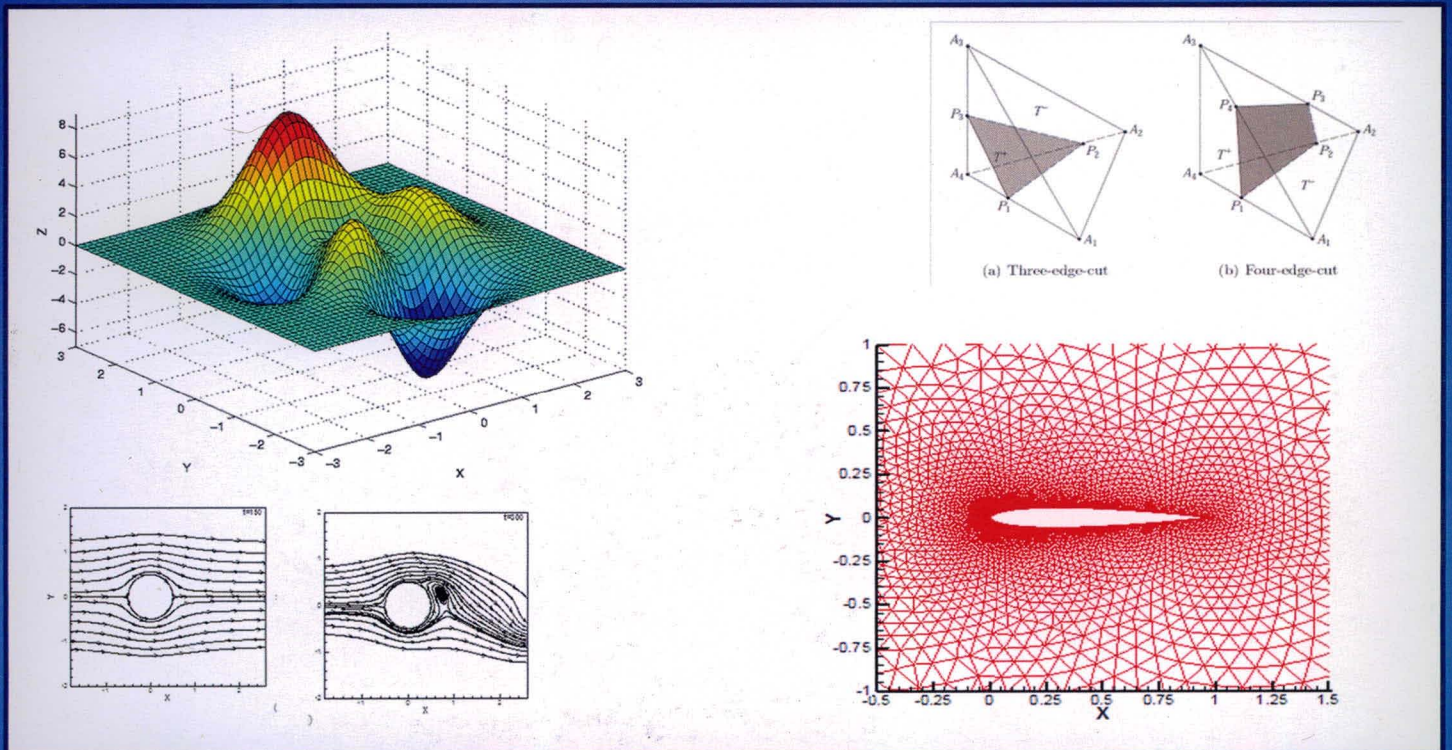


ADVANCED NUMERICAL TECHNIQUES IN ENGINEERING and SCIENCE



Editors

AHMAD TARIQ JAMEEL

WAQAR ASRAR



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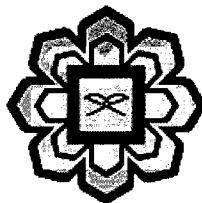
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CHAPTER 4

Unstructured Finite Volume For Two-Dimensional Navier Stokes Equations

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ABSTRACT

Unstructured finite volume for a two-dimensional Navier Stokes equations is analyzed in this chapter. Flow solver using upwind scheme on unstructured grids is developed. The procedure uses a dual mesh system to implement the finite volume method using conserved variables stored at the vertices of the mesh. The dual mesh system can adopt both triangular and quadrilateral elements. The second-order Roe flux-difference splitting scheme is implemented using MUSCL approach, where the primitive variables are extrapolated to cell faces to achieve second-order accuracy. Both Euler and Runge-kutta explicit time integration methods are implemented. Various limiters are incorporated into the code. These include Barth and Jespersion limiter, MinMod limiter, Van Albada limiter, Super-Bee limiter, and Venkatakrisnan limiter.

Different cases are considered to validate the code and to demonstrate the ability to solve practical aerospace flow problems. These cases are flow around NACA0012 airfoil, flow over a 10 degree Ramp, JPL nozzle. To assess accuracy and convergence of the code, the order of the scheme is estimated using the method of Roache and the grid convergence index is calculated. The computed results are compared with published numerical and experimental data. The effect of the choice of the limiter is demonstrated.