ADVANCED NUMERICAL
TECHNIQUES IN ENGINEERING AND
SCIENCE

Editors

AHMAD TARIQ JAMEEL
Department of Biotechnology Engineering

WAQAR ASRAR
Department of Mechanical Engineering

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## Contents

Preface ........................................................................................................................................ v

Chapter 1 Numerical Techniques – An Introduction ................................................................. 1
   Ahmad Tariq Jameel

Chapter 2 Numerical Simulation of a Simple Couette Flow in Matlab Using Explicit And Implicit Finite Difference Schemes
   Asif Hoda .................................................................................................................................. 7
   2.1 Introduction .......................................................................................................................... 8
   2.2 The Couette Flow Problem .................................................................................................. 9
   2.3 Numerical Simulation of Coutte Flow .................................................................................. 10
       2.3.1 Defining discrete points in the domain of interest ...................................................... 10
       2.3.2 Obtaining numerical approximation for the governing equation .............................. 12
       2.3.3 Reducing the differential equation to a set of algebraic equations ............................ 13
       2.3.4 Solution of the algebraic equation ................................................................................ 15
   2.4 Stability Analysis of Explicit and Implicit Schemes ............................................................ 20
   2.5 Conclusion .......................................................................................................................... 21
   Bibliography ............................................................................................................................. 22

Chapter 3 Comparison of a Pseudo-Spectral Method and an Implicit Finite Difference Scheme for the Solution of a Nonlinear Dynamic Problem
   Ahmad Tariq Jameel .................................................................................................................. 23
   3.1 Introduction ........................................................................................................................ 24
   3.2 Problem Definition ............................................................................................................. 24
   3.3 Numerical Method .............................................................................................................. 27
       3.3.1 Spectral Methods .......................................................................................................... 27
           3.3.1.1 Fourier Collocation Method ................................................................................. 29
Chapter 4  Unstructured Finite Volume for Two-Dimensional Navier Stokes Equations
Mounen Mohammed Idres and Ahmed Tawfeek El Taweel

4.1 Introduction
4.1.1 Finite volume method
4.1.2 Upwind schemes
4.1.3 High resolution schemes

4.2 Roe’s Upwind Scheme
4.2.1 Two-dimensional Navier Stokes equations
4.2.2 Node-Based finite volume method
4.2.3 Definition of control volume
4.2.4 Data structure of Node-Based method
4.2.5 Inviscid residual calculation
4.2.5.1 Roe’s flux difference splitting method
4.2.5.2 Data reconstruction for high-resolution scheme
4.2.5.2.1 TVD schemes
4.2.5.2.2 Barth and Jesperson scheme
4.2.5.2.3 MUSCL differencing approach

4.2.6 Viscous residual calculation
4.2.7 Time integration methods
4.2.7.1 Explicit Euler method
4.2.7.2 Explicit Runge-Kutta scheme

4.2.8 Boundary conditions
4.2.8.1 Inflow and outflow
4.2.8.2 Wall boundary conditions
4.2.8.2.1 Euler wall boundary conditions
4.2.8.2.2 Navier-Stokes wall boundary conditions
4.2.8.3 Far field boundary conditions

4.2.9 Convergence criterion

4.3 Code Validation
4.3.1 Discretization error estimation
4.3.1.1 Standard Richardson extrapolation
4.3.1.2 Smooth flow test case
4.3.2 Inviscid flow solver
4.3.2.1 Effect of element type on the Inviscid flow solver 73
4.3.2.2 Flow around NACA-0012 airfoil 78
   4.3.2.2.1 Subsonic case 81
   4.3.2.2.2 Critical case 83
4.3.2.3 Flow inside a duct with 10° ramp 85
4.3.3 Viscous flow solver 88
   4.3.3.1 Laminar flow over a flat plate 88
   4.3.3.2 Axisymmetric laminar flow inside JPL nozzle 90
4.4 Conclusion and Recommendations 94
References 96

Chapter 5  Higher-Order Compact Finite Difference Schemes 99
Yap Wen Jieun, Waqar Asra and, Mahmoud Khalid Mawlood

5.1 Introduction 100
5.2 Recent Works 102
5.3 Higher Order Compact Finite Difference Schemes 107
   5.3.1 Classification of Higher-Order compact schemes 108
      5.3.1.1 The Governing-Equation-Based scheme 108
      5.3.1.2 The Hermitian scheme 109
5.4 Higher Order Time Discretization Method 113
5.5 Application of HOC Schemes 115
   5.5.1 Viscous Burgers’ equation 115
      5.5.1.1 Lax-Wendroff approach 117
      5.5.1.2 Hermitian scheme 119
   5.5.2 HOC solution of Burgers’ equation on a clustered grid 122
      5.5.2.1 The Lax-Wendroff approach 124
      5.5.2.2 The Hermitian scheme 128
   5.5.3 Numerical results 129
      5.5.3.1 Numerical studies 134
      5.5.3.2 Numerical boundary conditions 136
      5.5.3.3 Grid clustering 145
5.6 Conclusion 150
References 151

Chapter 6  Higher Order Flux-Based Upwind Scheme for Compressible Flows 157
Nadeem Hasan, S. Mjaheed Khan, and Faisal Shameem

6.1 Governing Equations 158
6.2 The PVU Scheme: Origin and Development 158

xi
6.2 The PVU Scheme: Origin and Development

6.2.1 The PVU-M+ scheme

6.2.1.1 Estimation of inter-cell numerical particle velocity $u_{i+1/2}$ and convective transport property vector $\phi_{i+1/2}$

6.2.1.2 Estimation of $u_{i+1/2}$ and $\phi_{i+1/2}$ in the vicinity of shocks

6.3 Performance Assessment Criteria and 1-D Test Cases

6.3.1 One-dimensional Inviscid test cases

6.4 Multi-Dimensional Inviscid and viscous test cases

6.4.1 Supersonic inviscid flow past a forward facing step in a channel

6.4.2 Inviscid Shock-Vortex interaction

6.4.3 Two-dimensional inviscid compressible flow past a circular Cylinder

6.4.3.1 Low subsonic regime ($M_x = 0.2$)

6.4.3.2 Transonic flow ($M_x = 0.38-0.98$)

6.4.3.3 Supersonic flow at $M_x = 3.0$ and $M_x = 10.0$

6.4.4 Two-dimensional viscous compressible flow past a circular cylinder

6.4.4.1 ($M_x = 0.1, Re_x = 100$) flow past a circular cylinder

6.4.4.2 ($M_x = 0.7, Re_x = 2000$) flow past an adiabatic circular cylinder

6.5 Conclusion

References

Chapter 7

Finite Element Modelling of the Powder Compaction Process

Meftah Hrair and, Hedi Chtourou

7.1 Introduction

7.1.1 Powder metallurgy process

7.1.2 Powder metallurgy technology

7.1.3 Numerical simulation of powder compaction process

7.2 Finite Element Method

7.2.1 Large displacement formulation

7.2.2 Finite element discretization

7.2.3 Nonlinear iterative strategy

7.3 Powder Constitutive Model

7.3.1 Cap plasticity model
7.3.2 Numerical implementation 222
7.3.3 Integration of the Behaviour law 223
  7.3.3.1 Elastic prediction 224
  7.3.3.2 Plastic correction 225
  7.3.3.3 Elastic mode 226
  7.3.3.4 Tension mode 226
  7.3.3.5 Singular tension mode 226
  7.3.3.6 Shear mode 227
  7.3.3.7 Singular compression mode 228
  7.3.3.8 Cap mode 229
7.3.4 Updating the variables 230
  7.3.5 Derivation of Elastoplastic Tangent moduli 231
    7.3.5.1 Perfect plasticity 231
    7.3.5.2 Hardening plasticity (cap mode) 232
7.4 Application of cap plasticity model 234
  7.4.1 Introduction 234
  7.4.2 Determination of model parameters 234
    7.4.2.1 Elastic parameters 234
    7.4.2.2 Hardening law parameters 235
    7.4.2.3 Yield surfaces parameters 236
  7.4.3 Case studies 239
    7.4.3.1 Rotational-Flanged component 239
    7.4.3.2 Industrial gear 244
7.5 Conclusion 246
References 247

Chapter 8  
Introduction of Piecewise Virtual Fields Method for Solution of Inverse Problems  
Syed Muhammad Kashif  
253

8.1 Use of Full Field Data for Mechanical Characterization 254
  8.1.1 Introduction 254
  8.1.2 Solution of inverse problems using full field data 254
  8.1.3 Piecewise virtual fields method 257
  8.1.4 Conclusion 258
8.2 The Piecewise Virtual Fields Method in Plate Blending Problems 258
  8.2.1 Introduction 258
  8.2.2 Construction of the virtual fields 263
    8.2.2.1 Introduction 263
Chapter 9  **Immersed Finite Element Method (IFEM)**

_Raed Ismail Kafafy_

9.1 Introduction 296

9.1.1 Body-Fitting-Grid methods 296

9.1.2 Cartesian-Grid methods 297

9.1.3 Cartesian-Grid methods based on finite difference discretization

9.1.3.1 The immersed boundary method (IBM) 298

9.1.3.2 The level set method (LSM) 298

9.1.3.3 The smoothing method for discontinuous coefficients 299

9.1.3.4 The immersed interface method (IIM) 299

9.1.3.5 The embedded curved boundary method (ECB) 299

9.1.4 Cartesian-Grid methods based on finite element discretization

9.1.4.1 The partition of unity method (PUM) 299
9.1.4.1 The partition of unity method (PUM) 299
9.1.4.2 The extended finite element method (X-FEM) 300
9.1.4.3 The immersed finite element method (IFE) 300

9.2 Three-Dimensional IFE Method 301
9.2.1 The interface boundary value problem 302
9.2.2 Weak formulation of the field problem 303
9.2.3 A three dimensional IFE space 304
9.2.4 Intersection topology 306
  9.2.4.1 Special intersection topology 306
  9.2.4.2 Linear local nodal FE basis functions 307
  9.2.4.3 Linear local nodal IFE basis functions 308
  9.2.4.4 Three-edge cut element 309
  9.2.4.5 Four-edge cut element 312
9.2.5 Existence and uniqueness 315
9.2.6 Partition of unity and consistency with classical FEM 316

9.3 Building a 3d IFE field solver 319
9.3.1 Mesh generation 319
9.3.2 Mesh-Object intersection 319
9.3.3 Intersection topology classification 320
9.3.4 Assembly of the IFE system of equations 322
  9.3.4.1 Local assembler 322
  9.3.4.2 Global assembler 322
9.3.5 Integration rules 323
  9.3.5.1 Gaussian Quadratures 323
  9.3.5.2 Integration on interface elements 324
9.3.6 Sparse storage of the system matrix 326

9.4 Nonlinear IFE Solver 327
9.4.1 Gauss-Seidel iteration 327
9.4.2 Newton-Raphson iteration 328
9.4.3 Solution of the sparse linear/linearized system 329
9.4.4 Preconditioned-Conjugate Gradient (PCCG) solver 329
9.4.5 Preconditioners 329
9.4.6 Incomplete cholesky decomposition 330
9.4.7 Jacobi diagonal preconditioner 330
9.4.8 Hardwiring the IFE field solver 330
9.4.9 Hardwired local assembler 331

9.5 Numerical Examples 331
9.5.1 Results of Numerical Experiments Using IFE Method 331
  9.5.1.1 An interface problem with a spherical interface 332
  9.5.1.2 An interface problem with a Hemispherical 335
Chapter 10 Lower-Upper Symmetric-Gauss-Seidel (LU-SGS) Algorithm for Pseudo Compressibility Method

Ashraf Ali Omar

10.1 Introduction
10.2 Factorization and Relaxation
10.3 Three-Dimensional Incompressible Navier Stoke Equations
   10.3.1 Governing equations
   10.3.2 Transformation of the Governing equations
10.4 Pseudo-Compressibility Method
10.5 Space Discretization and Implicit Scheme
   10.5.1 Space Discretization
      10.5.1.1 Introduction
      10.5.1.2 Inviscid flux differencing
      10.5.1.3 Central differencing method
      10.5.1.4 Differencing of viscous flux terms
   10.5.2 Implicit scheme
      10.5.2.1 Introduction
      10.5.2.2 Pseudo-time discretization
      10.5.2.3 LU-SGS scheme
10.6 Initial and Boundary Condition
10.7 Pseudo-Time Step
10.8 Applications
   10.8.1 Incompressible viscous flow over a multi-element airfoil
      10.8.1.1 Studied model
      10.8.1.2 Results
         10.8.1.2.1 Convergence history
         10.8.1.2.2 Surface pressure
         10.8.1.2.3 Velocity profiles
         10.8.1.2.4 Lift Coefficients
   10.8.2 Incompressible vortical flows over a 3-D Tangent-Ogive cylinder
      10.8.2.1 Introduction
      10.8.2.2 Grid generation and boundary conditions
      10.8.2.3 Results and discussion
CHAPTER 3

Comparison of a Pseudo-spectral Method and an Implicit Finite Difference Scheme for the Solution of a Nonlinear Dynamic Problem

AHMAD TARIQ JAMEEL

Department of Biotechnology Engineering, Faculty of Engineering, International Islamic University Malaysia, Gombak, 50728 Kuala Lumpur, Malaysia.
atjameel@yahoo.com; atjameel@iiium.edu.my

ABSTRACT

The nonlinear dynamics of thin film flow on a solid plane is represented by the so called ‘equation of evolution (EOE)’ – a nonlinear fourth order partial differential equation. The extent of nonlinearity and stiffness of the EOE depends upon the nature of the different types of intermolecular and external forces considered in the body force term of the equation of motion. Further, a thin film subjected to various physico-chemical effects, such as thermocapillarity, marangoni flow, heat and mass transfer, and chemical reaction at the film interface also decide the extent of nonlinearity of the EOE. However, for the purpose of illustrating the numerical algorithm, we consider a general model of an ultrathin film of Newtonian liquid on a plane solid substrate subjected to apolar van der Waals and polar intermolecular forces expressed in terms of the second derivative of free energy, $\phi$ in the governing equation. The EOE was solved numerically for periodic boundary conditions as an initial value problem when the free surface of the film was initially subjected to a sinusoidal wave perturbation. Two different classes of numerical methods: a pseudo-spectral and an implicit finite difference discretization are used. The numerical results from the two techniques are compared. It is shown that the Fourier collocation (FC), a pseudo-spectral method is easy to implement for nonlinear problems with periodic boundary conditions. The computation time required