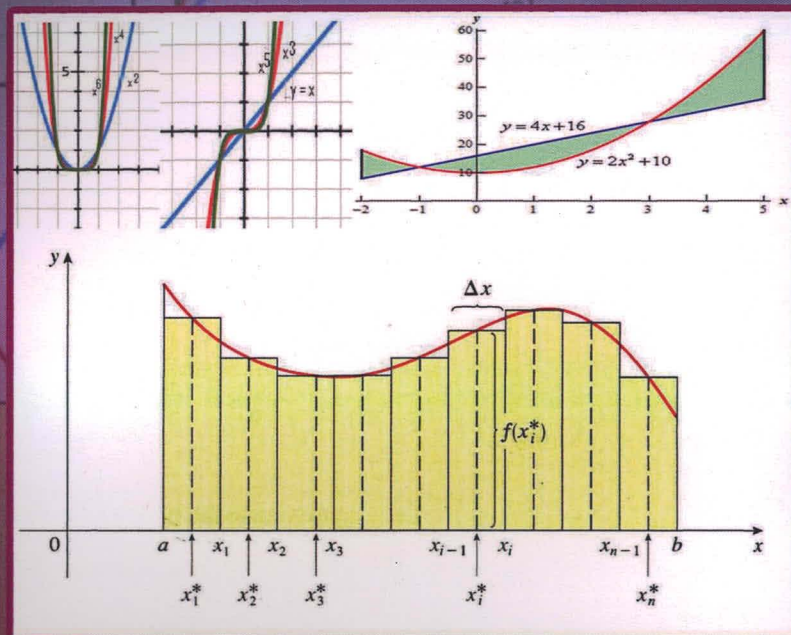


CALCULUS WITH SINGLE VARIABLE



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CHAPTER 5

APPLICATIONS OF DEFINITE INTEGRALS

Sellami Ali

In this section we investigate some of the applications of the definite integral by using it to compute area under curve, areas between curves, volumes of solids, and arc length and surface area. We define all of these as limits of Riemann sums of continuous functions on closed intervals, that is, as integrals, and evaluate these limits with calculus.

5.1 Areas under curve:

Here, we define and calculate areas of regions that lie under the graphs of functions. We want to find the area under a curve, $y = f(x)$ on the interval $[a, b]$. Initially we'll need to estimate the area under a curve. We will do this by dividing the region into rectangles and adding up the areas of rectangles, as follows. We divide the interval from a to b into n equal subintervals using the numbers $x_0, x_1, x_2, \dots, x_n$ with $a = x_0$ and $b = x_n$, as shown in figure . Since each subinterval is of equal length, we denote its length by Δx . Using the right-hand endpoint in each case, we construct rectangles with heights $f(x_0), f(x_1), \dots, f(x_n)$ and with width Δx (see figure).

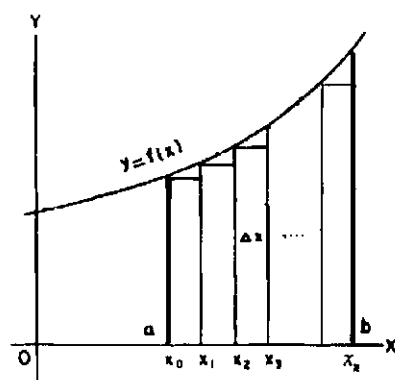


Figure 5.1

The area of each rectangle is the height times the width, so that the sum of the areas of all n rectangles is

$$f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x$$