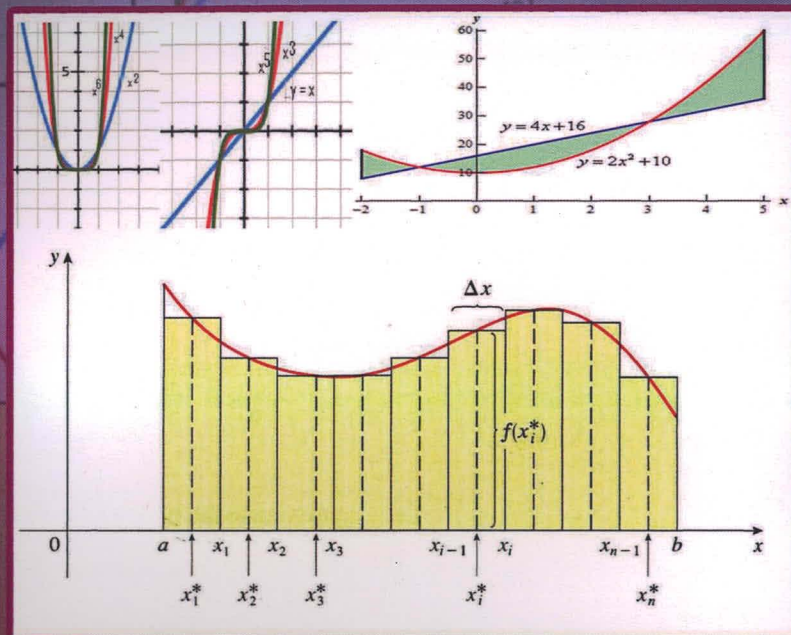


CALCULUS WITH SINGLE VARIABLE



M. S. H. CHOWDHURY

IUM PRESS
INTERNATIONAL ISLAMIC UNIVERSITY MALAYSIA



CALCULUS WITH SINGLE VARIABLE

EDITED BY
M. S. H Chowdhury



IIUM Press

Published by:
IIUM Press
International Islamic University Malaysia

First Edition, 2011
©IIUM Press, IIUM

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without any prior written permission of the publisher.

Perpustakaan Negara Malaysia

Cataloguing-in-Publication Data

M. S. H Chowdhury: Calculus with Single Variable

ISBN: 978-967-418-195-6

Member of Majlis Penerbitan Ilmiah Malaysia – MAPIM
(Malaysian Scholarly Publishing Council)

Printed by :
IIUM PRINTING SDN. BHD.
No. 1, Jalan Industri Batu Caves 1/3
Taman Perindustrian Batu Caves
Batu Caves Centre Point
68100 Batu Caves
Selangor Darul Ehsan

Table of Content

	PREFACE	i
	ACKNOWLEDGEMENTS	ii
<hr/>		
CHAPTER 0:	PRELIMINARIES	1-42
	By M. Azram and Messikh Azeddin	
<hr/>		
0.1	Number System and Inequalities	1
0.2	Functions	8
0.3	Trigonometric and Inverse Trigonometric Functions	14
0.4	Exponential and Logarithmic Function	28
0.5	Linear Transformations	31
	Exercise 0	38
CHAPTER 1:	LIMITS AND CONTINUITY	43-66
	By Gharib Subhi Mahmoud Ahmad and Raihan Othman	
<hr/>		
1.1	Introduction.	43
1.2	Computation of Limits	43
1.3	Calculating Limits Using the Limit Laws	48
1.4	Infinite Limits and Limits at Infinity	50
1.5	Tangent Limits and Derivatives	54
1.6	Continuity	60
	Exercise 1	64
CHAPTER 2:	THE DERIVATIVES	67-91
	By Jamal I. Daoud	
<hr/>		
2.1	The Slope of a Straight Line	67
2.2	Properties of the Slope of a Line	68
2.3	The Slope as a Rate of Change	70
2.4	The Slope of a Curve at a Point	71
2.5	The Derivative using the Definition	72
2.6	Differentiation Rules	75
	2.6.1 Derivative Constant Multiple Rule	76
	2.6.2 Derivative Sum and Difference Rule	76
	2.6.3 Derivative Product Rule	78
	2.6.4 Derivative Quotient Rule	79
2.7	Second-And Higher-Order Derivatives	80

2.8	Derivative of Trigonometric Functions	81
2.8.1	Derivative of the Sine Function	81
2.8.2	Derivative of Cosine Function	82
2.9	The Chain Rule	85
2.10	Implicit differentiation	87
	Exercise 2	89

CHAPTER 3: APPLICATIONS OF DIFFERENTIATION

By Zaharah Wahid

92-136

3.1	Linear Approximations and Newton's Method	92
3.1.1	Small Changes and Linear Approximations	92
3.1.2	Rates of Change	97
3.1.3	Newton's Method	99
3.2	Indeterminate Forms	102
3.2.1	Function Forms $0/0$, ∞/∞	102
3.2.2	L' Hospital's Rule or Theorem	102
3.3	Maximum and Minimum Values	105
3.3.1	Increasing and Decreasing Functions	105
3.3.2	Turning Points of a Curve	108
3.3.3	Maximum and Minimum Problem	112
3.4	First and the Second Derivative Tests and Curve Sketching	115
3.4.1	First derivative Test Increasing and Decreasing	116
3.4.2	Concavity and The second derivative Test	119
3.4.3	Second Derivative Test for Local Extrema	120
3.4.4	Distance, Velocity, and Acceleration	122
3.4.5	Instantaneous Velocity	123
3.4.6	Acceleration	124
3.5	Optimization Problems	127
3.5.1	Guideline for solving optimization	128
3.5.2	Applications	128
	Exercise 3	133

CHAPTER 4: INTEGRATION

By M. S. H. Chowdhury

137-168

4.1	Indefinite Integral	137
4.2	Sums and Sigma Notation	142
4.3	Area	146
4.4	Reimann Sum and Definite Integral	150

4.5	The Fundamental Theorem of Calculus	157
4.6	Integration by Substitution	162
	Exercise 4	166

CHAPTER 5: APPLICATIONS OF DEFINITE INTEGRALS By Sellami Ali **169-200**

5.1	Areas Under Curve	169
5.2	Area between Curves	171
5.3	Volume: Slicing Disks and Washers	176
5.4	Arc Length and Surface Area	189
5.5	Work	193
	Exercise 5	197

CHAPTER 6: INTEGRATION TECHNIQUES By M. S. H. Chowdhury **201-245**

6.1	Review of Integration Using Basic Formulas and Substitution	201
6.2	Integration by Parts	207
6.3	Trigonometric Techniques of Integration	214
6.4	Special Types of Integration	228
6.5	Integration of Rational Functions Using Partial Fractions	232
6.5	Improper Integrals	236
	Exercise 6	243

CHAPTER 7: INFINITE SERIES By Faiz A. Elfaki and M. Azram **246-266**

7.1	Sequences of Real numbers	246
	7.1.1 Definition of Sequence	246
	7.1.2 Limit of a Sequence	247
7.2	Series and Convergence	247
	7.2.1 Infinite Series	247
	7.2.2 Convergent and Divergent Series	248
	7.2.3 Geometric Series	249
	7.2.4 n th -Term Test for Divergence	250
7.3	The Integral Test and p -Series	251
	7.3.1 The Integral Test	251
	7.3.2 p - Series and Harmonic Series	252
7.4	Comparison of Series	252
	7.4.1 Direct Comparison Test	252

CHAPTER 4

INTEGRATION

M.S.H Chowdhury

4.1 Indefinite Integral

Indefinite integral is a process of summation, or the inverse process of the differentiation. If $f(x)$ be any anti-derivative of $F(x)$ then indefinite integral of $F(x)$ is defined by

$$\int F(x) dx = f(x) + C, \quad \text{where } C \text{ is an arbitrary constant (called integrating constant)}$$

where $F(x)$ is called the anti-derivative function of $f(x)$ or indefinite integral. $F(x)$ is called the integrand, dx is called the variable of integration.

Example 4.1-1: Prove that $f(x) = [\sin(x^4) + e^{x^3}]$ is an anti-derivative of

$$F(x) = 4x^3 \cos x^4 + 3x^2 e^{x^3}.$$

Solution:

We have, $f(x) = \sin x^4 + e^{x^3}$

$$\therefore \frac{d[f(x)]}{dx} = 4x^3 \cos x^4 + 3x^2 e^{x^3} = F(x).$$

Example 4.1-2: Find the anti-derivative of $F(x) = x^3$.

Solution:

$$f(x) = \int x^3 dx = \frac{1}{4}x^4$$

$$\therefore \frac{df}{dx} = \frac{d}{dx} \left[\frac{1}{4}x^4 \right] = x^3 = F(x).$$

Note that: By adding any constant C to the anti-derivative function $f(x)$ the derivative does not change i.e.,

$$\int F(x) dx = f(x) + C \quad \text{is a general solution where } C \text{ is an arbitrary constant.}$$