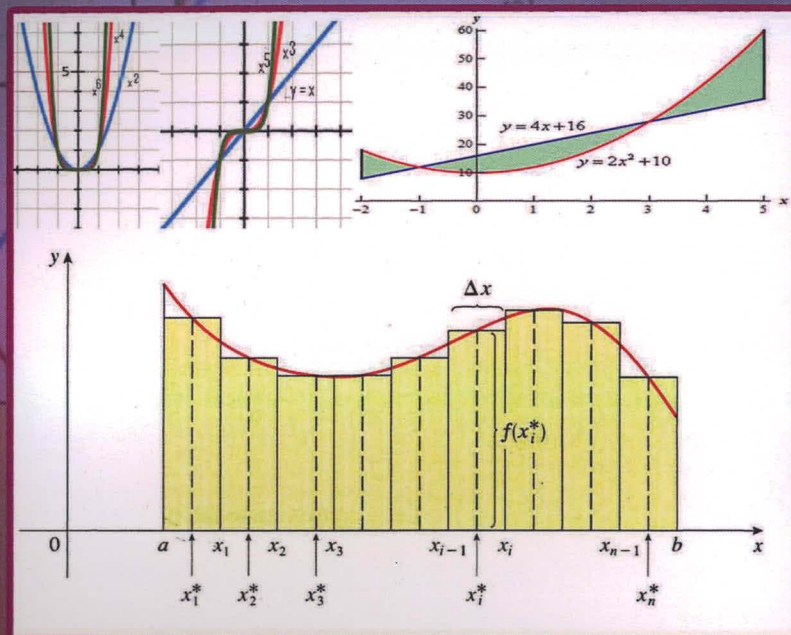


CALCULUS WITH SINGLE VARIABLE



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CHAPTER 1

LIMITS AND CONTINUITY

Gharib Subhi Mahmoud Ahmad and Raihan Othman

1.1 Introduction:

Calculus was invented primarily to treat the major scientific problems in physics of the 17th century. There were four major types of problems:

- i. To find the velocity and acceleration of a moving body at any instant given the formula for the distance covered as function of time.
- ii. Find the tangent to a curve.
- iii. Find the maximum or minimum value of a function.
- iv. Find the lengths of curves, such as the distance covered by a planet in a given period of time, the areas bounded by curves, volumes bounded by surfaces, centers of gravity of bodies, and the gravitational attraction that an extended body.

The concept of a limit was emerging naturally from the solutions of the above applied problems. Therefore there is no wonder that the concept of a limit is fundamental idea that distinguishes calculus from algebra and trigonometry. Limit is the milestone for derivative and integration.

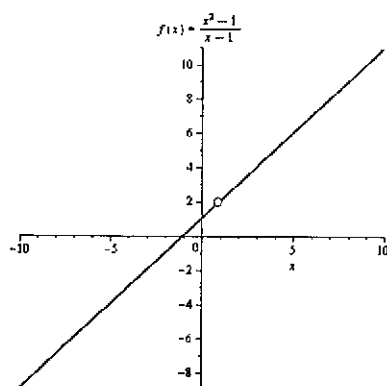
In this chapter we define the limit. We use the limits to describe the way a function f varies. The notation of limit gives a precise way to distinguish between the different behaviors of the functions. Furthermore, the geometric application of using limits to define the tangent to a curve lead at once to the important concept of the derivative of a function.

1.2 Computation of Limits

In this section we develop the notation of limit and illustrate the idea with some simple examples.

Example 1: Behavior of a function near a point

Let $y = f(x) = \frac{x^2 - 1}{x - 1}$. What happens to $f(x)$ as x get close to the value $x = 1$?



x	$f(x)$	x	$f(x)$
2	3	0	1
1.5	2.5	0.5	1.5
1.1	2.1	0.9	1.9
1.01	2.01	0.99	1.99
1.001	2.001	0.999	1.999
1.00001	2.00001	0.99999	1.99999