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INTERNATIONAL UNDERGRADUATE RESEARCH INNOVATION, INVENTION AND DESIGN

UNDERGRADUATES PIONEERING TOMORROW'S BREAKTHROUGH

E-BOOK OF EXTENDED ABSTRACTS

UNDERGRADUATES PIONEERING TOMORROW'S BREAKTHROUGH

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PREFACE

It is with great pleasure that we present this e-book of extended abstracts for the International Undergraduate Research, Innovation, Invention and Design (IURIID) organized by Universiti Teknologi MARA (UiTM) Cawangan Negeri Sembilan. Guided by the theme "Undergraduates Pioneering Tomorrow's Breakthrough," this publication reflects the energy, creativity, and determination of undergraduate students who are stepping forward as active contributors to knowledge, innovation, and design. The works showcased here represent a wide spectrum of disciplines and highlight the ability of young scholars to address real-world challenges, propose novel solutions, and demonstrate resilience in the pursuit of meaningful ideas. Each abstract captures not only academic effort but also the spirit of exploration, collaboration, and the courage to move beyond conventional boundaries. This collection further illustrates the importance of nurturing undergraduate research as a foundation for future breakthroughs and as a catalyst for personal and professional growth. We extend our heartfelt appreciation to the students for their dedication, the supervisors and mentors for their invaluable guidance, the reviewers for ensuring quality, and the organizing committee for their commitment in making this event a success. It is our hope that this e-book will serve as an inspiration to readers, encouraging continued innovation and affirming the vital role of undergraduates in pioneering tomorrow's breakthroughs.

Editors

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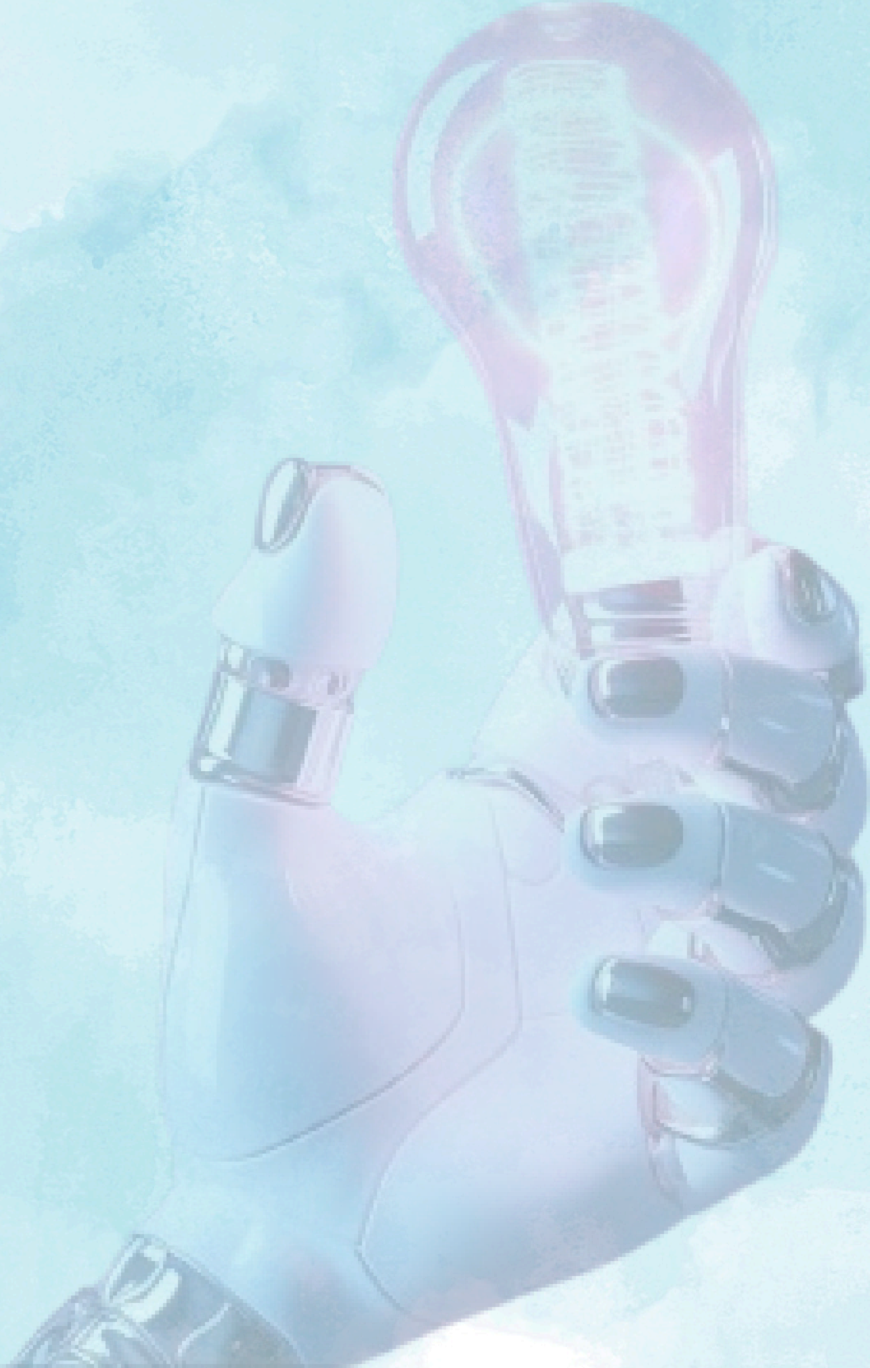
SCIENCE AND TECHNOLOGY



THE RATE OF CONVERGENCE OF b-BISTOCHASTIC QUADRATIC STOCHASTIC OPERATOR ON 1-DIMENSIONAL SIMPLEX

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ABSTRACT

This research studies the dynamics of a quadratic stochastic operator (QSO) called the b-bistochastic QSO defined on a 1-dimensional simplex. The QSO was first constructed on a 1-dimensional simplex before fixed-point analysis was conducted on the QSO. Under some conditions, the QSO is defined to be the b-bistochastic QSO. It is found that such b-bistochastic QSO converges to a unique fixed point at a constant rate.

Keywords: Quadratic Stochastic Operator, b-order Majorization, Hyperbolicity, Rate of Convergence

INTRODUCTION

The concept of the quadratic stochastic operator (QSO) was first introduced in 1924 through Bernstein's pioneering work on the theory of heredity and it is commonly used to describe the species time evolution in a biological system (Mukhamedov & Ganikhodjaev, 2015). In a biological system with n species or traits, the state space is represented by the simplex $S^{n-1} = \{x_1, x_2, x_3, \dots, x_n\}$. We denote $x^{(0)} = \left(x_1^{(0)}, \dots, x_n^{(0)}\right)$ as the probability distribution of the species where $x_i^{(0)}$ is the probability of species i. Let $p_{ij,k}$ be the probability that individual in the i^{th} and the j^{th} species interbreed to produce an individual of the species k. The probability distribution of the first generation, $x^{(1)} = \left(x_1^{(1)}, \dots, x_n^{(1)}\right)$ can be found by using a total probability,

$$x_k^{(1)} = \sum_{i,j=1}^{n} p_{ij,k} x_i^{(0)} x_j^{(0)}, \;\; k = 1, \dots, n. \tag{1}$$

The QSO, commonly denoted as V, describes the population evolution and dynamics across successive generations. Starting from the initial arbitrary state of probability distribution, $x^{(0)}$ evolves to the probability distribution of the first generation, $x^{(1)} = V\left(x^{(0)}\right)$ and continues to evolve to the second generation, $x^{(2)} = V\left(x^{(1)}\right) = V\left(V\left(x^{(0)}\right)\right) = V^2\left(x^{(0)}\right)$ and so on. Thus, each QSO describes the dynamics of probability distributions based on the parameters of $p_{ij,k}$ and the distribution of the current generation or state.

Formally, a QSO is defined on the $(n-1)$-dimensional simplex,

$$S^{n-1} = \{x = (x_1, \dots, x_2) \in R^n | x_i \geq 0, \sum_{i=1}^{0} x_i = 1, \forall i\}. \tag{2}$$

A general QSO $V: S^{n-1} \to S^{n-1}$ can now be defined as the following form:

$$V(x)_k = \sum_{i,j=1}^{n} p_{ij,k} x_i x_j, \;\; k = 1, \dots, n. \tag{3}$$

The probability coeffiecient $p_{ij,k}$ has the following properties:

(1) $p_{ij,k} \leq 0,$
(2) $p_{ij,k} = p_{ji,k},$
(3) $\sum_{i,j=1}^{n} p_{ij,k} = 1,\ k = 1, \dots, n.$

A general QSO can be characterized by equation (3) and the properties of its probability coefficient. Additional conditions and properties define each of the QSO class uniquely. For each $k \in \{1, \dots, n-1\}$, we define functional $U_k: R^n \to R$ by

$$U_k(x_1, \dots, x_n) = \sum_{i=1}^{k} x_i. \tag{4}$$

Based on equation (4), a new majorization is introduced called the *b*-order majorisation. Specifically, for any $x, y \in S^{n-1}$, we say x is said to be *b*-ordered by y if and only if $U_k(x) \leq U_k(y)$ for all $k \in \{1, \dots, n-1\}$, and is denoted as $x \leq^b y$. For any $x, y, z \in S^{n-1}$, the *b*-order majorization satisfies the following conditions:

1. $x \leq^b x,$
2. $x \leq^b y, y \leq^b \Leftrightarrow x = y,$
3. $x \leq^b y, y \leq^b z \Leftrightarrow x \leq^b z.$

Additionally, it also has the following properties:
1. $x \leq^b y \Leftrightarrow \alpha x \leq^b \alpha y, \forall \alpha > 0,$
2. $x \leq^b y, \alpha < \beta \Rightarrow \alpha x \leq^b \beta y.$

We provide important notions and information regarding the *b*-bistochastic QSO, canonical form and hyperbolicity as follows:

Theorem 1: Let V be a QSO. Then, if V satisfies $V(x) \leq^b x$ for all $x \in S^{n-1}$, then V is called a *b*-bistochastic QSO.

The following theorem by Mukhamedov and Embong (2015) describes the general properties of the *b*-bistochastic QSO.

Theorem 2: Let V be a *b*-bistochastic QSO defined on S^{n-1}, then the following statements hold:

1. $\sum_{m=1}^{k} \ \sum_{i,j=1}^{n} p_{ij,m} \leq kn,\ k \in \{1, \dots, n-1\},$
2. $p_{ij,k} = 0$ for all $i, j \in \{k+1, \dots, n\}$ where $k \in \{1, \dots, n-1\},$
3. $p_{nn,n} = 1,$
4. for every $x \in S^{n-1}$, one has:
   (i) $V(x)_k = \sum_{l=1}^{k} p_{ll,k} x_l^2 + 2\sum_{l=1}^{k} \ \sum_{j=l+1}^{n} p_{lj,k} x_l x_j$ where $k = \underline{1, n-1},$
   (ii) $V(x)_n = x_n^2 + \sum_{l=1}^{n-1} p_{ll,n} x_l^2 + 2\sum_{l=1}^{n-1} \ \sum_{j=l+1}^{n} p_{lj,n} x_l x_j,$
5. $p_{lj,l} \leq \frac{1}{2}$, for all $j \geq l+1,\ l \in \{1, \dots, n-1\}$

The behaviour of the *b*-bistochastic QSO was studied on a 1-dimensional simplex. Throughout this paper, only the 1-dimensional simplex will be considered, which is defined as follows.

Definition 1 [Mukhamedov & Ganikhodjaev, 2015]. For a population distribution with 2 species, from equation (2), the 1-dimensional simplex is defined as

$$S^1 = \{x = (x_1, x_2) \in R^2 | x_1, x_2 \geq 0, x_1 + x_2 = 1\}. \tag{5}$$

Definition 2 [Alligood et al., 2000]. Let $x \in R^n$ be a point of a function $f: R^n \to R^n$. Then, x is called a fixed point of the function f if $f(x) = x$.

If a point ξ exists for a QSO V such that $V(\xi) = \xi$, then x is said to be the fixed point of V. If an operator V has multiple fixed points, then the set of its fixed points is denoted as (V) as mentioned by Zada and Shah (2017). A fixed point ξ of a QSO can be further identified as hyperbolic or non-hyperbolic. From (3), we obtain the canonical form of the QSO as follows,

$$V(x)_1 = p_{11,1}x_1^2 + 2p_{12,1}x_1x_2 + p_{22,1}x_2^2. \tag{6}$$

By letting $p_{11,1} = a$, $p_{12,1} = b$ and $p_{22,1} = c$ be arbitrary coefficients such that $a, b, c \in [0,1]$ and expressing $x_2 = 1 - x_1$, we rewrite equation (6) as follows:

$$V(x)_1 = (a - 2b + c)x_1^2 + 2(b - c)x_1 + c. \tag{7}$$

Thus, let $a = 1, c = 0$ and from (7), we obtain the following

$$V(x)_1 = 2bx - 2bx^2. \tag{8}$$

For our case, we consider the case where $a \neq 1$, that is $a \in [0,1)$ and $b \neq \frac{1}{2}$. Let $\lambda = V'(\xi)$ where ξ is the fixed point. The hyperbolicity coefficient can be determined by the following,

$$\lambda = 2\,(a - 2b + c)\xi + 2(b - c). \tag{9}$$

According to Mukhamedov and Ganikhodjaev (2015), a fixed point ξ is called hyperbolic fixed point if $|\lambda| \neq 1$ and non-hyperbolic otherwise. In the hyperbolic case, ξ can be further identified as either an attracting fixed point or a repelling fixed point. The dynamical behaviour of the hyperbolic fixed point ξ of the operator V is determined by the hyperbolicity λ.

Theorem 3: If $|\lambda| < 1$, then ξ is said to be an attracting fixed point. If $|\lambda| > 1$, then ξ is said to be a repelling fixed point.

The behavior of the hyperbolic fixed point ξ can also be determined by using the discriminant, Δ of equation (9).

Lemma 1: If $0 < \Delta < 4$, then ξ is an attracting fixed point and if $4 < \Delta < 5$, then ξ is a repelling fixed point.

The interval $(0,5)$ contains all possible values of Δ, as proven by Mukhamedov and Ganikhodjaev (2015). From Shahidi (2013), we have the following,

Definition 3. A QSO V is said to be a regular QSO if for any arbitrary point of the QSO, $V^m(x)$ exists. If otherwise, then V is said to be a non-regular QSO.

Next, from Mukhamedov and Embong (2015), we have the following,

Proposition 1. Let V be a *b*-bistochastic QSO on S^1 defined on (5). Then, $\xi = (0,1)$ is one of the fixed points.

RESULTS AND DISCUSSION

We obtain that $a = 1, c = 0$ and from **Proposition 1**, the attracting fixed point is $\xi = (0,1)$. Thus, from equation (9), we have that

$$\lambda = 2(1 - 2b)(0) + 2b = 2b. \tag{10}$$

The rate of convergence of an operator describes the rate at which the operator converges to its attracting fixed point (Ahlberg et al., 2021) as its iteration goes unbound. The rate is determined by λ defined in (9). Specifically, for a QSO defined on S^1, the rate of convergence of the QSO is described in the following theorem (Mukhamedov and Ganikhodjaev, 2015).

Theorem 4: Let $V(x) = (a - 2b)x^2 + 2bx$ be a b-bistochastic QSO defined on S^1 in (8) and let $a \in [0,1]$ and $b \in [0, \frac{1}{2})$. Then, for any given initial point x_0, the operator converges to the fixed point $\xi = (0,1)$ at the rate $C(x_0)(2b)^m$.

In this case, $C(x_0)$ is any constant that depends on the initial condition x_0. Note that this rate of convergence of the b-bistochastic QSO on S^1 is specific for when $b < \frac{1}{2}$ only.

CONCLUSION

The rate of convergence of the b-bistochastic QSO on 1-dimensional simplex S^1, was obtained, showing that the QSO converges to the attracting fixed point $\xi = (0,1)$ at a rate of $C(x_0)(2b)^m$. The result is verified for the case of $b < \frac{1}{2}$. This work presents preliminary findings on the convergence rate. A more detailed analysis including the stochastic matrix approach and ergodicity properties has been submitted for publication in the Malaysian Journal of Fundamental and Applied Sciences (MJFAS).

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