

Characteristics of nerve graph: a view from graph terminologies

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ABSTRACT - In this study, the characteristics of nerve graph are investigated through graph theory. This study addresses the problem of identifying and characterizing nerve graph from the perspective of simple, degree, bipartite, adjacency and cycles terminologies. Employing the proof-by-cases method, this article systematically analyses the characteristics of the nerve graph and reveals distinct features based on its structural configurations. This research significantly contributes to graph theory field by enhancing the understanding of nerve graph and paving the way for further research and practical applications.

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1. INTRODUCTION

Graph theory is a fundamental field of mathematics that is essential in a variety of disciplines, including biology, computer science, and network analysis [1–2]. By abstracting real-world problems into graphical forms, graph theory has significantly simplified numerous intricate problems. Its core concept is the representation of these issues as graphs, which are composed of vertices (or nodes) and edges (or connections). This approach enables more accessible analysis and the discovery of solutions [3]. A graph provides a comprehensive framework for representing a wide range of systems and relationships, including biological systems and social networks [4]. The study of graphs encompasses a diverse range of categories, including simple graphs, bipartite graphs, and those characterised by specific properties such as degree, cycles, paths, and adjacency relations [5–6]. Bipartite graphs are particularly important for understanding relationships between two distinct sets of vertices, which is relevant to network design and matching problems [7–8]. Other notable types of graphs include planar graphs, which can be drawn on a plane without edge intersections, and weighted graphs, in which edges are assigned weights representing costs or distances [9–10].

Nerve graphs occupy a distinctive and specialised position among the various varieties of graphs. A nerve graph represents the structure of the human nervous system, consisting of nerve vertices connected by nerve edges [3]. Islamic cupping points for specific medical conditions are represented by these nerve vertices, which are connected by nerve edges to form a coherent and functional structure [3]. This unique definition highlights the significance of nerve graphs in modelling complex biological structures and their interactions, especially in traditional medical practices such as cupping therapy [11].

The primary objective of this study is to investigate the structural characteristics of nerve graphs through the rigorous lens of graph theory. Specifically, it aims to identify and describe nerve graphs by analysing their structural properties such as whether they are bipartite or non-bipartite, their adjacency relations, the vertex with the maximum degree, and whether these graphs contain cycles [12]. This study adopts a proof-by-cases approach to systematically analyse the characteristics of nerve graphs, thereby uncovering distinct structural features and paving the way for further research and practical applications [13].

This research is significant not only for its contribution to theoretical graph theory but also for its potential applications in biological and medical sciences. Understanding the structural properties of nerve graphs could lead to improved models of nervous system function and enhance the effectiveness of treatments in traditional medicine, such as Islamic cupping [14]. Furthermore, this study contributes to the broader field of graph theory by exploring various terminologies and concepts such as simple graphs, degree, bipartite graphs, and cycles in the context of nerve graphs, thereby providing valuable insights into their properties [15]. Additionally, the research opens avenues for future investigations, particularly into other types of biological networks or the refinement of existing graph models [16].

In summary, this study seeks to bridge the gap between the theoretical aspects of graph theory and the practical applications of nerve graphs. By offering a comprehensive analysis of the characteristics of nerve graphs, it not only

enhances the understanding of these specialised graphs but also lays the groundwork for future research and applications in related fields [17].

2. METHODOLOGY

The methodology involves defining key concepts and outlining the approach for analysing nerve graphs. This section presents the relevant definitions and approach used for the study. Twelve definitions and one theorem are provided in the preliminaries section.

2.1 Preliminaries

Definition 1 [5]: Graph, G

A graph is a pair consisting of two sets where the first set is the non-empty set of vertices and the other set is the set of edges. A graph can be denoted as $G = (V, E)$ where V is the set of vertices and E is the set of edges connecting between vertices.

The vertices and edges will be the cupping point and human nerves, respectively. The definition of the approach method is then provided.

Definition 2 [5]: Degree of vertex

The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by $\deg(v)$.

Definition 3 [18]: Proper colouring

A colouring of a simple graph is the assignment of a colour to each vertex of the graph so that no two adjacent vertices are assigned the same colour.

Definition 4 [19]: Chromatic number, $\chi(G)$

The chromatic number, $\chi(G)$ of the graph, G is the least number of colours required for colouring a graph.

The chromatic number is determined by counting the total amount of colour utilized after assigning a colour to each vertex. The colour with the smallest element will be selected as guide for making decision.

Definition 5 [5]: Bipartite

A simple graph G is called bipartite if its vertex set v can be partitioned into two disjoint sets v_1 and v_2 such that every edge in the graph connects a vertex in v_1 and a vertex in v_2 (so that no edge in G connects either two vertices in v_1 or two vertices in v_2). When this condition holds, we call the pair (v_1, v_2) a bipartition of the vertex set v of G .

In addition, there exists a theorem that relates to bipartite graphs as in Theorem 1.

Theorem 1 [20]: Bipartite

A graph G is bipartite if and only if it contains no odd cycles.

Bipartite graph is one where vertices can be neatly divided into two groups such that each edge links vertices from different groups, ensuring that no two vertices within the same group are directly connected. Next, the acyclic and odd cycle definition will be defined.

Definition 6 [21]: Acyclic

An acyclic graph is a graph without cycles.

Definition 7 [22]: Odd Cycle

A cycle with an odd number of vertices is called an odd cycle.

In other words, if a vertex is started at and the cycle is traversed, the starting vertex will be returned to after an odd number of vertices has been visited. Next, Nerve Vertex, Nerve Edge, Nerve Graph and Nerve Cupping definition will be defined.

Definition 8 [3]: Nerve Vertex, NV

Based on the human nerve system, the nerve vertex, NV represents an Islamic cupping point for a specific medical condition.

Definition 9 [3]: Nerve Edge, NE

The nerve edge, NE is an edge that connects the nerve vertex, NV to form a graph based on the human nerve system.

Definition 10 [3]: Nerve Graph, $N(G)$

The term nerve graph, or NG refers to a collection of nerve vertices, NV joined by nerve edges, NE . In other words, the nerve graph appears as follows $NG = (NV, NE)$.

Definition 11: Nerve Cupping, NC

A nerve cupping, designated as NC , represents sets of specific cupping points that need to be cupped during Islamic cupping therapy. In other words, the sets of nerve cupping are considered the optimum number of cupping therapy offered.

Definition 12: Optimal Nerve Cupping, ONC

The nerve cupping, NC is considered the optimal solution, ONC when it consists of the solution with the minimum number of nerve vertices through the process of proper colouring method.

Ideally, the medical cupping can be modelled mathematically as depicted in Table 1.

Table 1. The relation between cupping points and the mathematical model

Cupping Point	Mathematical Model
Islamic cupping point	Nerve vertex, NV
Human nerve system	Nerve edge, NE
Cupping points system	Nerve graph, NG
Set of nerve vertex that will be cupped	Nerve cupping, NC
Nerve cupping that has minimum number of nerve vertices	Optimal nerve cupping, ONC

The Table 1 establishes a relation between Islamic cupping therapy and a mathematical model, aligning elements of cupping therapy with graph theory concepts. Here, cupping points are likened to nerve vertices, the human nerve system to nerve edges, and the cupping point system to a nerve graph. A group of targeted nerve vertices forms a "nerve cupping," creating a network-like model that represents the cupping process in terms of interconnected nodes and edges.

In the next section, the results of the analysis will be presented. These results highlight the key findings and insights derived from the methodology applied to the nerve graphs. In addition, the results implicate the context of the study's objectives. These results employ a proof-by-cases approach to analyse the nerve graph's characteristics. This methodology involves systematically examining different scenarios to understand the properties of nerve graphs. The following propositions were explored:

Proposition 1: If n is a nerve vertex, NV in a nerve graph, NG possesses maximum degree then n is one of the elements in optimal nerve cuppings, ONC .

Proposition 1 demonstrates that a nerve vertex in a nerve graph. With maximum degree must be one of the optimal nerve cuppings.

Proposition 2: If a nerve graph, NG is simple and non-bipartite, then its chromatic number is at least 3.

Proposition 2 investigates the chromatic number of a nerve graph by considering whether it is simple and non-bipartite.

Proposition 3: Let NG be a nerve graph, then none of the element in the set of nerve cupping, in NG are adjacent.

Proposition 3 analyses the adjacency of nerve cupping point within the nerve graph.

Each proposition is examined through various cases to validate the results and draw conclusions about nerve graphs' structural features. This section discusses the key findings regarding nerve graphs, focusing on their degree, chromatic number, cycle structure, and the adjacency of nerve cupping point. The results provide insights into the characteristics and applications of these specialised graphs.

In the study of nerve graphs, a critical property of vertices is their degree, which refers to the number of edges connected to them. This proposition explores the relationship between a vertex possessing the maximum degree and its classification as a nerve cupping. Specifically, it aims to prove that if a vertex in a nerve graph has the highest degree, it must be classified as one of the nerve cuppings.

Proposition 1: If n is a nerve vertex, NV in a nerve graph, NG possesses maximum in degree, then n is one of the elements in optimal nerve cuppings, ONC .

Proof: By contradiction, we need to prove that if n is not one of the optimal nerve cupping, the result will not hold. Let n be a vertex that possess maximum degree, implementing proper colouring method, the first colour will be assigned as maximum as it can on the prospect nerve graph, NG . This implies the vertex with minimum degree has a high possibility to be coloured. Since the optimum of the nerve cupping will be selected by the lowest number of elements of the results, hence the vertex with high degree has a major chance to odd cycle be selected which contradict with our assumption. Thus, the proposition hold.

The proof demonstrates that in the case of a nerve graph with a single vertex, the vertex automatically holds the maximum degree and qualifies as a nerve cupping. In cases with multiple vertices, a contradiction argument shows that a vertex with the maximum degree must be a nerve cupping due to its incomparable degree to others. Thus, the proposition is validated that a vertex with the maximum degree in a nerve graph is indeed one of the nerve cuppings.

To further elucidate proposition 1, a practical example will be examined. This example 1 will facilitate the visualization of how the theoretical results of the proposition are applied in a real-world context. By analysing a specific nerve graph, it will be demonstrated how vertices with the maximum degree are identified as nerve cuppings, thereby reinforcing the validity of the proposition.

Example 1

Figure 1 shows the nerve graph, NG for back pain. 10 nodes, b_i $1 \leq i \leq 10$ representing nerve vertices (NV) used by practitioners to treat back pain. The nerve edges (NE) are the connections between these 10 nodes. Using graph theory, the connection of the nerve vertices generates a nerve graph, NG . The ideal number of nerve vertices for medicinal purposes is determined by applying the concept of proper colouring to the nerve graph NG . Table 2 show the degree for each vertex while Figure 2 shows the process of applying proper colouring to the nerve graph NG .

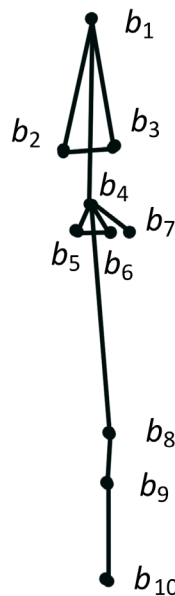


Figure 1. Nerve graph, NG for back pain

Table 2. Degree for each vertex for back pain

Vertex	Degree
b_1	3
b_2	2
b_3	2
b_4	5
b_5	2
b_6	2
b_7	1
b_8	2
b_9	2
b_{10}	1

By proposition 1, all these clearly displayed in the Table 2, b_4 is one of the nerve cuppings because possesses the maximum degree.

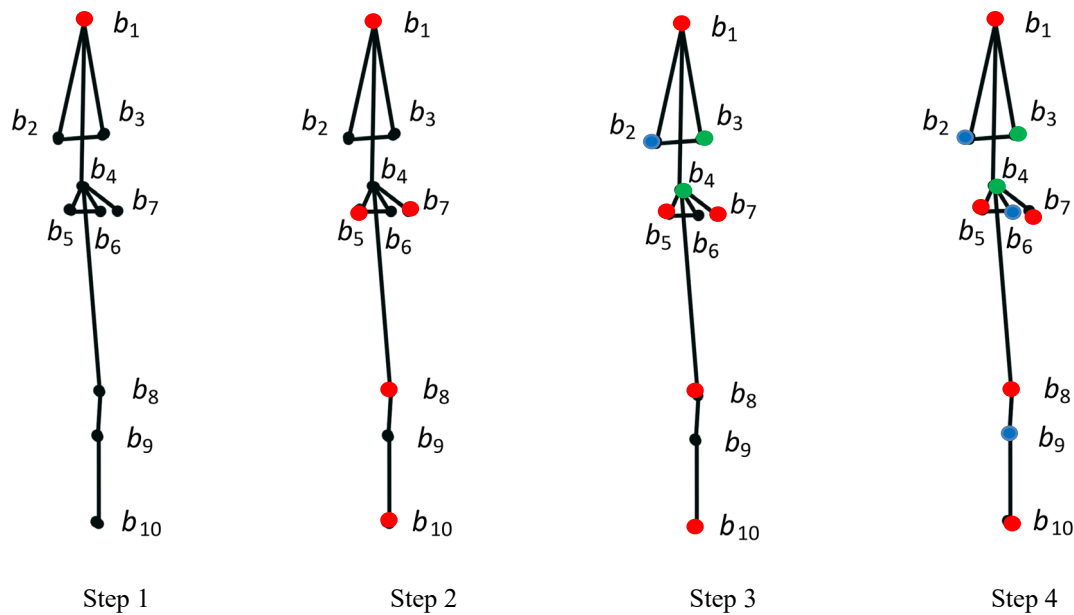


Figure 2. Steps for colouring the retrieved figure with proper colouring methods for nerve graph of back pain

As proper colouring concept applied, first, a vertex is selected randomly. In the first step, vertex b_1 is chosen and coloured red. Next, all vertices not adjacent to b_1 , such as b_5 , b_7 , b_8 , and b_{10} , are also coloured red. In proper colouring concept, when colouring the vertices, the first colour chosen should be placed as the maximum as it can on the nerve graph. Subsequent to b_2 , b_3 , and b_4 are adjacent to b_1 , automatically b_2 is assigned a different colour, blue. Meanwhile, b_3 and b_4 have been assigned green, respectively. All the colouring processes are repeated until all the vertices are coloured, as shown in Step 4. The chromatic numbers that were acquired are $\chi(G) = 3$, at the end of the process. To achieve accurate result, the trial-and-error method is employed during the proper colouring process. This method is shown in Table 3, where two different patterns are recorded.

Table 3. Results of proper colouring for nerve graph ng (back pain)

Colour	Result 1	Result 2
Red	5	3
Blue	3	4
Green	2	3

According to definition 12, Result 1 is the optimal nerve cupping, among the two potential results in Table 3.2 because the minimum number degree of set nerve vertices is two, through the process of proper colouring method. The details are as follows:

Result 1: $\chi(G) = 3$

- Red : 5 nerve vertices, $\{b_1, b_5, b_7, b_8, b_{10}\}$
- Blue : 3 nerve vertices, $\{b_2, b_6, b_9\}$
- Green : 2 nerve vertices, $\{b_3, b_4\}$

As the Results 1, vertices in green will be one of the nerve cuppings is b_3 and b_4 . Based on the results, b_4 that have the highest of degree is one of the nerve cuppings. Its conclude that by proposition 1 is true.

Determining the chromatic number is essential in understanding the colouring properties of various types of graphs. In the next proposition, focuses on nerve graphs, which are specialized graphs representing Islamic cupping point and their connections in the human nervous system [3]. Specifically, we aim to prove that if a nerve graph is simple and non-bipartite, then its chromatic number is at least 3.

Proposition 2: If a nerve graph is simple and non-bipartite, then its chromatic number is at least 3.

Proof. Let NG be a nerve graph which is simple and non-bipartite. NG can be a disconnected or connected graph and if NG is connected then either NG contains an odd cycle or NG does not contain an odd cycle. Then there are three cases need to be considered:

Case 1: NG is disconnected

Case 2: NG is connected and contains an odd cycle

Case 3: NG is connected and does not contain an odd cycle.

Case 1: NG is disconnected

Let $NG_1, NG_2, \dots, NG_k$, be the connected components of NG . Given all connected components are non-bipartite, then by Definition 2.5, it can't be partitioned into two disjoint non-empty set. Therefore, the chromatic number of NG , $\chi(NG_i) \geq 3$.

Case 2: NG is connected and contains an odd cycle

Suppose NG is connected and contains an odd cycle. Then by theorem 1, NG cannot be bipartite thus $\chi(NG) \geq 3$.

Case 3: NG is connected and does not contain an odd cycle

Since NG is not bipartite, clearly NG contain odd cycle.

This concludes the proof by cases that if a nerve graph is simple graph and non-bipartite, then its chromatic number is at least 3.

In all three cases whether the graph NG is disconnected, connected with an odd cycle, or connected without an odd cycle, it has been demonstrated that the chromatic number $\chi(NG)$ is at least 3. This result highlights an essential characteristic of simple and non-bipartite nerve graphs, that required a minimum of three colours for the chromatic number. To illustrate the practical application of proposition 2, example 2 is examined. Figure 3 is presented, and the application of the proposition to determine its chromatic number is demonstrated.

Example 2

Figure 3 shows the result of proper colouring method for nerve graph, NG for back pain. The nerve graph illustrates a simple and non-bipartite graph with 10 vertices, b_i , $1 \leq i \leq 10$ representing nerve vertices, NV used by practitioners to treat back pain. The presence of two odd cycle, specifically the triangle formed by b_1, b_2, b_3 and b_4, b_5, b_6 makes this graph non-bipartite. As a result, the chromatic number of this graph is at least 3, as adjacent vertices cannot be coloured with the same colours without violating the colouring rules.

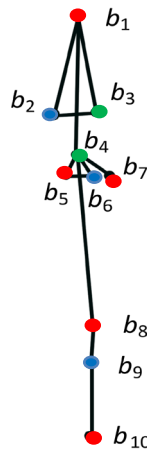


Figure 3. Result of colouring the retrieved figure with proper colouring methods for nerve graph of back pain

Understanding the relationship between cycles and cupping point in nerve graphs reveals the important structural aspects. A cupping point in a nerve graph is a vertex with significant degree properties related to the graph's cycles. In addition, it practically implies the nerve vertex with the maximum degree automatically selected as one of the nerve cuppings. Moreover, the chromatic number demonstrated that practitioner could apply at least 3 set of basic Islamic cupping point. Next, proposition 3 is discussed.

Proposition 3: Let NG be a nerve graph, then none of the element in the set of nerve cupping, in NG are adjacent.

Proof. Suppose NG is a nerve graph. Let v_1, v_j is the element in nerve cuppings, NC . By definition of proper colouring, it is not possible for the element of nerve cuppings NC to be adjacent in a connected graph. This concludes the proposition 3.

The results across all scenarios confirm that nerve cupping point in a nerve graph are non-adjacent. This non-adjacency is significant as it ensures that cupping point are distinct in their positions and interactions within the graph, reflecting their independent functional roles. To further illustrate the concept of non-adjacency among nerve cuppings in a nerve graph, example 3 will be considered. By examining Figure 3, the theoretical results of proposition 3 can be seen to manifest in practice. This example will provide a concrete visualization of the claim that no two nerve cuppings in a nerve graph are adjacent, thereby reinforcing the validity of the proposition.

Example 3

The Figure 3 above illustrates a nerve graph NG with two cycles, highlighting the nerve cupping points b_3 and b_4 in green. As demonstrated, these cupping points are not adjacent to each other, satisfying proposition 3 that none of the elements in the set of nerve cupping point in NG are adjacent.

4. CONCLUSIONS

The findings demonstrate that a vertex with the highest degree is one of the optimal nerve cuppings. Besides, if a nerve graph NG is simple, non-bipartite, and contains n cycles, then NG has at least n cupping point, each of which is non-adjacent to the others. Furthermore, the chromatic number of G is at least 3, if it is non-bipartite. This chromatic number is consistent with the presence of at least $n \geq 3$. By offering a more comprehensive examination of graph structures and their attributes, these discoveries contribute to the broader field of graph theory and improve our comprehension of nerve graphs. These findings could be further investigated in practical scenarios to investigate their implications and applications.

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AUTHOR CONTRIBUTIONS

Nur Atikah Aziz (Conceptualisation; Proving, Resources; Writing- original draft), Yuhani Yusof (Conceptualisation; Proving; Writing- review & editing), Nor Haniza Sarmin (Proving; Writing- review & editing), Hazulin Mohd Radzuan (Conceptualisation; Writing- review & editing), Viska Noviantri (Resources; Writing- review & editing).

DECLARATION OF ORIGINALITY

The authors declare no conflict of interest to report regarding this study conducted.

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