



Evaluation of derivative in damping in the Newtonian limit for non-planar wedge

Shamitha^{a,*}, Asha Crasta^a, Sher Afghan Khan^b

^a Department of Mathematics, M.I.T.E, Moodabidri & Affiliated To VTU, Belgavi, Karnataka 574225, India

^b Department of Mechanical Engineering, Faculty of Engineering, IIUM, Gombak Campus, Kuala Lumpur 68100, Malaysia

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ABSTRACT

The current work derives the analytical expression for damping derivative of a non planar wedge when γ tends to one and Mach number tends to infinity. Ghosh's developed strip theory is utilized to derive the expression of damping derivative. With regard to a variety of geometrical and flow characteristics, the current theory can forecast the damping derivatives of a non planar wedge. Prior to performing exhaustive calculations and trial research, it is vital to know about these damping derivatives in order to freeze and arrive at the geometrical and kinematic similarity parameters. The ongoing technique, which is exceptionally useful during the plan stage, predicts the damping subordinates in pitch for a flat wedge effortlessly. In the Newtonian limit, the equations derived for stability derivatives become precise. The pivot position is found to influence the damping derivative directly.

Additionally, it has been noted that at high angles of attack, the centre of pressure shifts significantly from the leading edge to the trailing edge. Consequently, according to the viewpoint of stability, this behavior may be utilized to stabilize the aeronautical vehicle. Therefore, in this case, the expression for the damping derivative is non-linear, the findings have been affected accordingly. However, the behaviour is linear up to a fifteen degree angle of attack before the pattern becomes non-linear.

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1. Introduction

In-depth research has been done on unstable supersonic/hypersonic aerodynamics for small angles of attack and moderate supersonic/hypersonic Mach numbers. For the case of an oscillating wedge, Hui [1] and Carrier [2] provided exact solutions, while Hui [3] provided an exact solution for an oscillating flat plate. They are equally applicable to all supersonic Mach numbers, arbitrary angles of attack, and wedge angles as long as the shock waves are connected to the body's leading edge. In a supersonic/hypersonic flow, the shock wave may be attached to or divided from the leading edges of an oscillating triangular wing depending on the flight Mach number, angle of attack, ratio of gas specific heats, and swept-back angle of the wing. Both the detached shock sce-

nario in hypersonic flow and the connected shock case were explored by Hui and Hemdan [4], and both prove true for medium attack angles. The strip theory was used in Hui et al.'s [5] investigation into the stability of an oscillating flat plate wing with any plan form placed at a particular mean angle of attack in a supersonic/hypersonic stream. In the Newtonian framework, the strip theory is precise. Since the local Newtonian flow is actually two-dimensional, fluid particles do not interact with one another. The Ghosh theory [6] has been expanded by Crasta and Khan to include planar wedges in Hypersonic and Supersonic Flow ([10]&[11]). Crasta and Khan have further developed this theory for Delta wings in hypersonic and supersonic flows with straight and curved leading edges ([7;8]), respectively. Crasta and Khan have also researched the stability derivatives for straight leading [9] and curved leading edges ([12], [13] & [14]) in the Newtonian limit. In this study, hypersonic planar similitude was employed, and relationships were found for the stability derivatives of pitch and roll for a planar wedge in the Newtonian limit.

* Corresponding author.

E-mail address: shamithashetty092@gmail.com (Shamitha).

1.1. Analysis

Consider a Planar wedge as shown in fig.

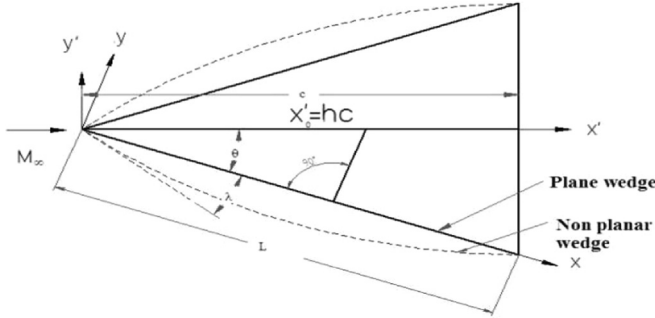


Fig.A. Planar wedge pivot position transfer from $x = 0$ to x'
The Stiffness and damping derivative is given by

$$-C_{m_x} = \frac{\gamma + 1}{2M_\infty^2 \cos^3 \theta} (I_1 + I_2 + I_3)$$

$$I_1 = M_\infty^2 \left[K \sin 2\theta - \frac{2\lambda \cos 2\theta}{3} \right]$$

$$I_2 = \left[\left\{ -M_\infty \frac{(z)^3}{3d} \right\} \left\{ K \cos \theta - (K \lambda \sin \theta + \cos \theta) \left(\frac{5c - 3z}{5d} \right) + \lambda \sin \theta \left(\frac{15z^2 - 42cz + 35c^2}{3d^2} \right) \right\} \right]_{z=c-d}^{z=c+d}$$

$$I_3 = \left[\left\{ \frac{M_\infty z^3}{2\lambda \cos \theta} \right\} \left\{ -K \sin \theta \cos \theta + \left(\sin \theta \cos \theta - \lambda K (\cos^2 \theta + \cos 2\theta) \right) \left(\frac{z-3c}{3d} \right) + \lambda (\cos^2 \theta + \cos 2\theta) \left(\frac{3z^2 - 10cz + 15c^2}{15d^2} \right) \right\} \right]_{z=c-d}^{z=c+d}$$

In the preceding equations

$$K = 1 - 2h \cos^2 \theta$$

$$c = \left(\frac{4}{\gamma + 1} \right)^2 + M_\infty^2 \sin^2 \theta$$

$$d = \lambda M_\infty^2 \sin 2\theta$$

In the Newtonian limit M_∞ tends to infinity and γ tends to unity. Therefore,

$$-C_{m_x} = \frac{1}{\cos^2 \theta} \lim_{\gamma \rightarrow 1} \left(\frac{I_1}{M_\infty^2} + \frac{I_2}{M_\infty^2} + \frac{I_3}{M_\infty^2} \right)$$

$$M_\infty \rightarrow \infty$$

$$= \frac{1}{\cos^2 \theta} \left[K \sin 2\theta - \frac{2\lambda \cos 2\theta}{3} + \left\{ \frac{1}{3\lambda \sin 2\theta} \right\} \left\{ (\sin^2 \theta + \lambda \sin 2\theta)^{\frac{3}{2}} - (\sin^2 \theta - \lambda \sin 2\theta)^{\frac{3}{2}} \right\} \right. \\ \left. \left\{ K \cos \theta - (K \lambda \sin \theta + \cos \theta) \frac{6}{5} + \frac{24}{35} \frac{\sin^2 \theta}{\sin 2\theta} \right\} + \left\{ \frac{1}{2\lambda \cos \theta} \right\} \left\{ (\sin^2 \theta - \lambda \sin 2\theta)^{\frac{1}{2}} - (\sin^2 \theta + \lambda \sin 2\theta)^{\frac{1}{2}} \right\} \left\{ -K \sin \theta \cos \theta - \frac{2}{3} (\sin \theta \cos \theta - \lambda K (\cos^2 \theta + \cos 2\theta)) + \frac{8 \sin^2 \theta}{15 \sin 2\theta} (\cos^2 \theta + \cos 2\theta) \right\} \right]$$

Damping derivatives is given by,

$$-C_{m_q} = \frac{\gamma + 1}{2M_\infty \cos^3 \theta} (J_1 + J_2 + J_3)$$

$$J_1 = M_\infty \left[K^2 \sin \theta + \left(\frac{\sin \theta - 2K \lambda \cos \theta}{3} \right) \right]$$

$$J_2 = \frac{-2Z^3}{3d} \left[\frac{K^2}{4} + K \left(\frac{5c - 3z}{10d} \right) + \frac{15z^2 - 42cz + 35c^2}{140d^2} \right]_{z=c-d}^{z=c+d}$$

$$J_3 = \left[\left\{ \frac{-Z^3}{\lambda \cos \theta} \right\} \left\{ \left(\frac{K^2 \sin \theta}{4} \right) - k (\sin \theta - \lambda k \cos \theta) \left(\frac{z-3c}{6d} \right) + (\sin \theta - 4\lambda k \cos \theta) \left(\frac{3z^2 - 10cz + 15c^2}{60d^2} \right) + \lambda \cos \theta \left(\frac{5z^3 - 21z^2c + 35z^2c^2 - 35c^3}{70d^3} \right) \right\} \right]_{z=c-d}^{z=c+d}$$

In the preceding equation,

$$K = 1 - 2h \cos^2 \theta$$

$$c = \left(\frac{4}{\gamma + 1} \right)^2 + M_\infty^2 \sin^2 \theta$$

$$d = \lambda M_\infty^2 \sin 2\theta$$

Therefore the damping derivatives in Newtonian limit is given

$$-C_{m_q} = \frac{1}{\cos^3 \theta} \lim_{\gamma \rightarrow 1} (J_1 + J_2 + J_3)$$

$$M_\infty \rightarrow \infty$$

$$= \frac{1}{\cos^3 \theta} \left[\left\{ K^2 \sin \theta + \left(\frac{\sin \theta - 2K \lambda \cos \theta}{3} \right) \right\} + \left\{ \frac{2}{3\lambda \sin 2\theta} \right\} \left\{ (\sin^2 \theta + \lambda \sin 2\theta)^{\frac{3}{2}} - (\sin^2 \theta - \lambda \sin 2\theta)^{\frac{3}{2}} \right\} \left\{ \frac{K^2}{4} + K \left(\frac{3}{5} \right) + \frac{6}{35\lambda} \frac{\sin^2 \theta}{\sin 2\theta} \right\} + \left\{ (\sin^2 \theta + \lambda \sin 2\theta)^{\frac{1}{2}} - (\sin^2 \theta - \lambda \sin 2\theta)^{\frac{1}{2}} \right\} \left\{ -K \sin \theta \cos \theta - \frac{2}{3} (\sin \theta \cos \theta - \lambda K (\cos^2 \theta + \cos 2\theta)) + \frac{8 \sin^2 \theta}{15 \sin 2\theta} (\cos^2 \theta + \cos 2\theta) \right\} \right]$$

1.2. Results and discussion

The derived analytical equations for a non planar wedge in supersonic/hypersonic flow's damping derivative in pitch was expanded in order to provide closed form solutions for pitch damping derivatives for a non planar wedge in the Newtonian limit; where, the Mach number will tend to infinity and the Specific heat ratio gamma (γ) will tend to one, When the stability derivatives in this case are taken into account, it is important to note how they will change as the geometric and flow parameters change The fluctuation of damping derivatives with the non-dimensional pivot point is depicted in Fig. 1. The figure shows that the damping derivatives drop linearly with pivot location for various angles of incidence, and the trend is as anticipated. The damping derivative is also found to increase linearly as the angle of attack increases, which is extremely true and follows the expected trend. The figure shows that the value of the damping derivatives is gradually increasing. There is a forty eight percent rise in the damping

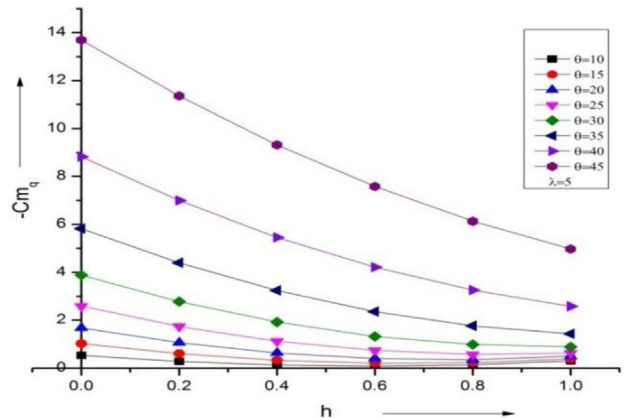


Fig. 1. Fluctuation of the damping derivative with the pivot position in the newtonian limit.

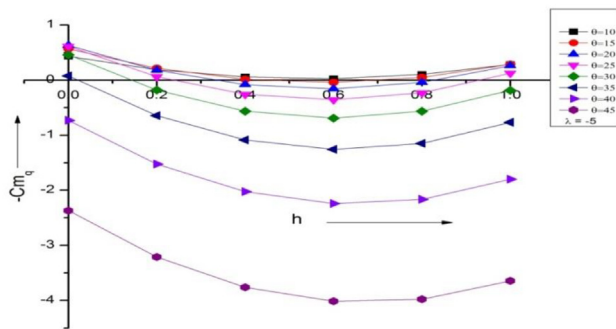


Fig. 2. Fluctuation of the damping derivative with the pivot position in the Newtonian Limit.

derivative when the angle of attack is raised from five to 10 degrees. Similarly, for the range between ten degrees to fifteen degrees the increase in the damping derivatives is around fifty percent. Further, it is found that for range between fifteen degrees to twenty degrees there is a fifty six percent increase in the damping derivative. Similarly, for the angle of incidence in the range twenty to twenty five and twenty five to thirty degrees the increase in the damping derivative is observed to be in the range sixty two to sixty five percent. (See Fig. 2).

One more finding from the figure is that as the angle of attack increases, the centre of pressure continuously shifts to the wedge's rear position. Due to this continuous shift of the center of pressure, The aerodynamic vehicles could be stabilized using this technique of continuously increasing the The need for a large stabilizing surface and the angle of incidence could be avoided. or in some cases the use of the tail fins can be completely avoided.

From these results It is clear that the values of the damping derivatives continuously increase as the angle of attack increases at $h = 0.0$, it is also observed that for lower angles of incidence the curve becomes flat for the pivot positions in the range from $h = 0.3$ to 0.6 , and this trend may be due to the variation in the pressure distribution on the non-planar surface. It is found that for attack direction within the range five degrees to ten degrees, ten degrees to fifteen degrees, fifteen degrees to twenty degrees, twenty degrees to twenty five degrees, and twenty five degrees to thirty degrees the increase in the damping derivatives are hundred percent, fifty percent, forty eight percent, thirty five percent, forty three percent, respectively.

1.3. Conclusion

When the shockwave is attached to the leading edge, the current theory is correct. Viscous effects and secondary wave reflections are not taken into consideration. Because they are calculated in the Newtonian limit, in which the specific heat ratio γ tends to unity and Mach numbers tend to infinity, the results show that the Mach number has no effect on damping derivatives. The damping derivative is discovered to linearly vary with the pivot location, just as was the case in our earlier findings for situations at low subsonic, supersonic, and hypersonic Mach numbers. For the whole scope of the current study, the damping derivative grows linearly as the angle of attack increases. Additionally, it has been noted that at high angles of attack, the centre of pressure shifts significantly from the leading edge to the trailing

edge. Therefore, from the perspective of static stability, this behaviour might be used to stabilize the aeronautical vehicle. In the instance of the damping derivative, the results have been reflected in the non-linear expression for the damping derivative.

CRedit authorship contribution statement

Shamitha: Data curation, Writing – review & editing, Writing – original draft, Software. **Asha Crasta:** Conceptualization, Methodology, Supervision, Validation. **Sher Afghan Khan:** Visualization, Investigation.

Data availability

The data that has been used is confidential.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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