The performance of Leland option pricing models in the presence of transaction costs: Evidence from the Australian index option market

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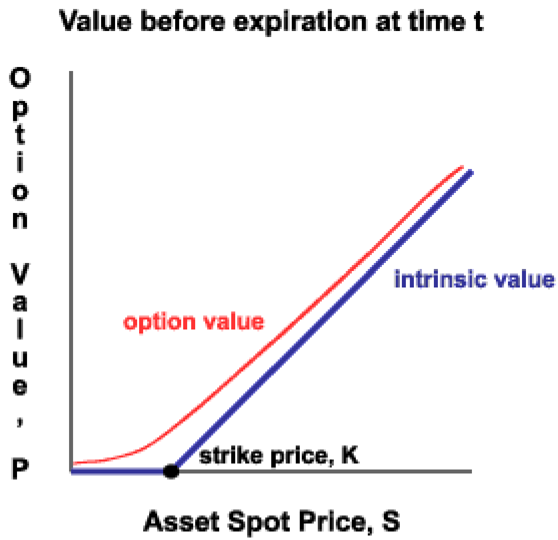

Introduction

The paper aims to examine the performance of option pricing model with transaction costs based on Australian index option data, specifically:

- Consider the mispricing errors for systematic tendencies related to option moneyness and time to maturity
- To investigate the performance of option pricing model in relation to different rebalancing intervals
- To identify the factors that influence the performance of option pricing model

Review of option pricing models with transaction costs

Author	Method/Approach
Leland (1985)	***Perfect replication***, modify BSM model via adjusted volatility, single options
Merton (1989)	***Perfect replication***, two period binomial model
Boyle & Vorst (1992)	***Perfect replication***, several periods binomial model
Bensaid et al. (1992) & Edirisinghe, Naik & Uppal (1993)	***Super-replication*** that dominates the option payoff at lower initial cost
Hoggard, Whalley & Wilmott (1994)	Work with same assumptions as Leland, valid not only on single options but portfolio of options
Davis & Clark (1994) and Soner, Shreve & Cvitanic (1995)	Proved that the least expensive super-replication strategy: initially buy asset and hold until maturity
Perrakis & Lefoll (2000), Perrakis & Lefoll (2004)	Extend Bensaid et al. – American calls and puts respectively
Leland (2007)	Provide adjustments to Leland (1985) Incorporate initial trading costs of trading with the assumptions of initial portfolio consists of all cash and all stock positions

Review of option pricing models with transaction costs (contd.)

Author	Method/Approach
Hodges & Neuberger (H&N)(1989)	**Utility maximisation** – Stochastic optimal control problem
Davis, Panas & Zariphopoulou (1993)	Modify H&N to include proportional costs to amount of stocks traded
Clewlow & Hodges (1997)	Modify H&N to include fixed and proportional costs
Whalley & Wilmott (1997)	Addressed the computational problem of H&N by providing asymptotic analysis
Barles & Soner (1998)	Extend H&N – provide alternative analysis
Constantinides & Zariphopoulou (1999) and Constantinides & Perrakis (2002)	Worked on stochastic dominance approach with investors' risks modelled as an increasing concave utility function
Zakamouline (2006)	Extend Davis et al. – provide alternative to asymptotic analysis – approximation strategy

Research gap

- Limited existing empirical studies on performance of various option pricing models has been focused on US S&P 500 index options
- Other empirical studies on comparing performances of different pricing and hedging strategies with transaction costs were focused on simulated results
- No empirical studies on option pricing model with transaction costs based on S&P/ASX 200 index option

Research gap (contd.)

- Some disadvantages of utility-maximisation approach:
  - lack of closed-form solution and calculations of the optimal hedging are time consuming (Zakamouline 2006, 2008);
  - difficult to handle and impractical because numerical computations are time consuming (Atkinson & Alexandropoulos 2006);
  - slow to compute, usually result in three- or four-dimensional free boundary problems (Whalley & Wilmott 1999);
  - investor’s risk must be specified, may not valid in reality; and
  - market must be continuously monitored (Gregoriou, Healy & Ioannidis 2007).

- Motivation for using Leland (1985) approach
  - development of Leland (1985) model
  - the unresolved questions of whether Leland’s method can be used to price options with realistic trading costs and rebalancing frequencies
  - has a closed-form solution and does not depend on investor’s risk

Option pricing models used

a) Black-Scholes-Merton (BSM) model

$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2) \qquad (1)$$

and

$$p = K e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1) \qquad (2)$$

where

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r - q + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad d_2 = \frac{ln\left(\frac{S_0}{K}\right) + \left(r - q - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

N(x) is the cumulative probability distribution function for a standardised normal distribution;
c and *p* are the European call and European put price respectively;
S_0 is the price of the underlying asset;
K is the strike price;
r is the continuously compounded risk-free rate and q is the dividend yield rate
σ is the underlying asset price volatility;
T is the time to maturity of the option.

Option pricing models used (contd.)

b)* *Leland models

- ***Leland (1985) model***

Leland formula for a call and a put:

$$c = S_0 e^{-qT} N\left(d_1^*\right) - K e^{-rT} N\left(d_2^*\right) \qquad (3)$$

$$p = K e^{-rT} N\left(-d_2^*\right) - S_0 e^{-qT} N\left(-d_1^*\right) \qquad (4)$$

Similar to BSM formula except that d_1 and d_2 are based on adjusted volatility for trading costs

$$\sigma^* = \sigma \left(1 + \frac{k\sqrt{\frac{2}{\pi}}}{\sigma\sqrt{\Delta t}} \right)^{1/2}$$

σ is the underlying risky asset standard deviation
Δt is the rebalancing interval (trading frequency)
k is the transaction cost rate

Option pricing models used (contd.)

- ***Leland (2007) models***

i) Leland (2007) cash model

Assuming initial portfolio consists of all cash positions:

$$c = \left(1 + \frac{k}{2}\right) S_0 e^{-qT} N\left(d_1^*\right) - K e^{-rT} N\left(d_2^*\right) \tag{5}$$

ii) Leland (2007) stock model

Assuming initial portfolio consists of all stock positions:

$$c = \left(\frac{k}{2}\right) S_0 e^{-qT} + \left(1 - \frac{k}{2}\right) S_0 e^{-qT} N\left(d_1^*\right) - K e^{-rT} N\left(d_2^*\right) \tag{6}$$

Formulas (1), (3), (5) and (6) are used.

Data

- Uses data on S&P/ASX 200 index call option (XJO index call option), S&P/ASX 200 index levels and Australian 90-day Bank Accepted Bill interest rate
- Daily index option data: trading date, expiration date, closing price, strike price and trading volume for each trading option
- Daily closing index levels
- Prior to 2nd April 2001, excessive movements due to changes of underlying asset of S&P/ASX 200 index option
- Sample period: 2nd April 2001 to 27th July 2005

Data (contd.)

Sampling procedure

- Apply some filter rules to remove offending daily option prices
  - ➢ Remove observations that do not satisfy minimum value arbitrage contraints (Bakshi, Cao & Chen 1997; Sharp & Li 2010)

$$C(\tau) \geq max\,[0, S_0 - KB(\tau)]$$

C(τ) is the price of call maturing in τ periods (years)
K is the exercise price of the option
S_0 is the initial index level
r is the risk-free rate of return
B(τ) is the current price of a $1 zero coupon bond with the same maturity as the option

➢ Remove observations that have less than 6 days to maturity (Bakshi, Cao & Chen 1997)

➢ Remove observations with exercise price of zero - LEPOs

Data (contd.)

Table 1. Sample Properties of S&P/ASX 200 Index Options

Moneyness (m)	Time to maturity in days (T)			
S/K	T < 30 (short-term)	30 ≤ T < 90 (medium-term)	≥ 90 (long-term)	Total
OTM (m < 0.97)	5.84 pts	19.52 pts	44.37 pts	29.45 pts
	287	*2007*	*1789*	*4083*
ATM (0.97 ≤ m < 1.03)	33.84 pts	64.99 pts	102.95 pts	64.40 pts
	1605	*3718*	*1216*	*6539*
ITM (m ≥ 1.03)	261.01 pts	194.26 pts	247.79 pts	226.15 pts
	162	*209*	*47*	*418*
Total	47.84 pts	54.16 pts	70.84 pts	57.60 pts
	2054	*5934*	*3052*	*11040*

Sample Average

S/K	Maturity (days)	Volume	Open interest	Series traded per day
0.98	70.65	76.06	815.84	7.00

Methodology

- Performance of option pricing models for a given transaction costs under various rebalancing intervals
  - Examine pricing errors (using RMSE) with and without taking into account the option's moneyness and time to maturity
  - Compare and contrast results with BSM model
- Apply different rebalancing intervals: quarterly, monthly, weekly and daily

Methodology (contd.)

Variables

- Time to maturity
  - T is measured by the number of trading days between the day of trade and the day immediately prior to expiry days divided by the number of trading days per year. There are 252 trading days per year (Hull 2003). Expiry date is not taken into account
- Realised volatility
  - Use realised volatility to determine the return standard deviation
- Daily return of index:

$$R_i = ln\left(\frac{S_i}{S_{i-1}}\right)$$

S_i is the index level

R_i is the log-return on the *i*th day during the remaining life of the option

$\overline{R}_t$ is the mean of daily log-returns during the period t

Methodology (contd.)

Therefore, the annualised realised volatility is

$$\sigma_{r,t} = \sqrt{\frac{252}{n-2}\sum_{i=2}^{n}\left(R_{i,t} - \overline{R}_t\right)^2}$$

- Risk-free interest rate

  Use Australian 90-day Bank Accepted Bill rate as proxy for risk-free interest rate. Convert interest rates to continuous compounding risk-free interest rates

- Transaction costs

  Trading fee is 0.2% (Do, 2002; Do & Faff, 2004)

- Dividends

  Use continuous compounded dividend yield of 3.65% (Reserve Bank Bulletin (2003), Reserve Bank Australia Statistics)

- Rebalancing intervals

  Apply rebalancing intervals: quarterly, monthly, weekly and daily

Methodology (contd.)

- To examine whether pricing errors exhibit pricing biases – moneyness, time to maturity, adjusted volatility and risk-free interest rate biases
- Carry out regression analysis of model pricing errors

$$E_n(t) = \beta_0 + \beta_1 \frac{S(t)}{K_n} + \beta_2 T_n + \beta_3 \sigma_n + \beta_4 r_n + \varepsilon$$

where

$E_n(t)$ is the n-th call option percentage pricing error on day *t*

β_i 's are the coefficients of independent variables

$\frac{S(t)}{K_n}$ is the moneyness

T_n is the time to maturity

σ_n is the adjusted volatility

r_n is the risk-free interest rate

Findings

Overall pricing errors

Model	Pricing errors (PE)	Underpricing (UP) versus Overpricing (OP)
BSM	RMSE **(highest)**	Underpricing > overpricing Most frequent underpricing
Leland (1985)	RMSE ↓as rebalance frequency ↑	Underpricing > overpricing
Leland (2007) cash	RMSE **(lowest)** RMSE ↓as rebalance frequency ↑	Underpricing > overpricing
Leland (2007) stock	RMSE ↓as rebalance frequency ↑	Underpricing > overpricing Least frequent underpricing

Findings (contd.)

F-tests results for overall pricing errors

F-tests at the 5% level - to determine whether there are any significant differences between two values of RMSE for any two consecutive rebalancing intervals. Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively

Model		**F stat**	**F critical**	**Decision**
Leland (1985)	$RMSE_Q$ versus $RMSE_M$	1.0348	1.0380	No significant differences
	$RMSE_M$ versus $RMSE_W$	1.0783	1.0380	Significant differences
	$RMSE_W$ versus $RMSE_D$	1.1019	1.0380	Significant differences
Leland (2007) cash	$RMSE_Q$ versus $RMSE_M$	1.0348	1.0380	No significant differences
	$RMSE_M$ versus $RMSE_W$	1.0667	1.0380	Significant differences
	$RMSE_W$ versus $RMSE_D$	1.0670	1.0380	Significant differences
Leland (2007) stock	$RMSE_Q$ versus $RMSE_M$	1.0302	1.0380	No significant differences
	$RMSE_M$ versus $RMSE_W$	1.0641	1.0380	Significant differences
	$RMSE_W$ versus $RMSE_D$	1.0619	1.0380	Significant differences

Findings (contd.)

Pricing errors for OTM call options

Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively.

Model	Short-term		Medium-term		Long-term	
	PE	UP vs OP	PE	UP vs OP	PE	UP vs OP
BSM	RMSE **(lowest)**	UP > OP Most frequent underpricing	RMSE **(highest)**	UP > OP Most frequent underpricing	RMSE **(highest)**	UP > OP Most frequent underpricing
Leland (1985)	RMSE ↑as rebalance frequency ↑	UP > OP But OP > UP for D	RMSE ↑as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	UP > OP
Leland (2007) cash	RMSE ↑as rebalance frequency ↑	UP > OP But OP > UP for D	RMSE ↑as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	UP > OP
Leland (2007) stock	RMSE **(highest)** RMSE ↑as rebalance frequency ↑	OP > UP	RMSE **(lowest)** RMSE ↑as rebalance frequency ↑	UP > OP Least frequent underpricing	RMSE **(lowest)** RMSE ↓as rebalance frequency ↑	UP > OP Least frequent underpricing

Findings (contd.)

Pricing errors for ATM call options

Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively.

Model	Short-term		Medium-term		Long-term	
	PE	UP vs OP	PE	UP vs OP	PE	UP vs OP
BSM	RMSE **(highest)**	UP > OP Most frequent underpricing	RMSE **(highest)**	UP > OP Most frequent underpricing	RMSE **(highest)**	UP > OP Most frequent underpricing
Leland (1985)	RMSE ↓as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	UP > OP
Leland (2007) cash	RMSE (**lowest)** RMSE ↓as rebalance frequency ↑ (from Q→W)	UP > OP	RMSE **(lowest)** RMSE ↓as rebalance frequency ↑	UP > OP	RMSE **(lowest)** RMSE ↓as rebalance frequency ↑	UP > OP Least frequent underpricing
Leland (2007) stock	RMSE ↓as rebalance frequency ↑ (from Q→W)	UP > OP at Q, M and W OP>UP at D	RMSE ↓as rebalance frequency ↑	UP > OP Least frequent underpricing	RMSE ↓as rebalance frequency ↑	UP > OP

Findings (contd.)

Pricing errors for ITM call options

Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively.

Model	Short-term		Medium-term		Long-term	
	PE	UP vs OP	PE	UP vs OP	PE	UP vs OP
BSM	RMSE **(highest)**	UP > OP Most frequent underpricing	RMSE **(highest)**	UP > OP Most frequent underpricing	RMSE **(highest)**	All UP
Leland (1985)	RMSE ↓as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	All UP
Leland (2007) cash	RMSE (**lowest**) RMSE ↓as rebalance frequency ↑	UP > OP Least frequent underpricing	RMSE **(lowest)** RMSE ↓as rebalance frequency ↑	UP > OP Least frequent underpricing	RMSE **(lowest)** RMSE ↓as rebalance frequency ↑	All UP
Leland (2007) stock	RMSE ↓as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	UP > OP	RMSE ↓as rebalance frequency ↑	All UP

Findings (contd.)

Regression analysis for overall pricing errors

Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively. Yes indicates the pricing errors exhibit pricing bias and No indicates the pricing errors exhibit no pricing bias.

Pricing bias	**BSM**	**Leland (1985)**	**Leland (2007) cash**	**Leland (2007) stock**
Moneyness	Yes	**No for W**	**No for W**	Yes
Time to maturity	Yes	Yes	Yes	Yes
Adjusted volatility	Yes	Yes	Yes	Yes
Risk-free interest rate	Yes	Yes	Yes	Yes

Findings (contd.)

Regression analysis for pricing errors for OTM call options

Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively. Yes indicates the pricing errors exhibit pricing bias and No indicates the pricing errors exhibit no pricing bias.

Pricing bias	**BSM**			**Leland (1985)**			**Leland (2007) cash**			**Leland (2007) stock**		
	Short	Med	Long	Short	Med	Long	Short	Med	Long	Short	Med	Long
Moneyness	Yes	Yes	Yes	Yes	Yes	**No for W**	Yes	Yes	**No for W**	**No for D**	Yes	Yes
Time to maturity	Yes	Yes	**No**	Yes	**No for D**	Yes	Yes	**No for W, D**	Yes	Yes	Yes	Yes
Adjusted volatility	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Risk-free interest rate	**No**	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Findings (contd.)

Regression analysis for pricing errors for ATM call options

Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively. Yes indicates the pricing errors exhibit pricing bias and No indicates the pricing errors exhibit no pricing bias.

Pricing bias	**BSM**			**Leland (1985)**			**Leland (2007) cash**			**Leland (2007) stock**		
	Short	Med	Long	Short	Med	Long	Short	Med	Long	Short	Med	Long
Moneyness	Yes	Yes	**No**	Yes	**No for Q, M**	**No for Q, M**	**No for W**	**No for Q, M**	**No for Q, M**	Yes	Yes	**No for Q**
Time to maturity	Yes	Yes	Yes	Yes	Yes	Yes	**No for Q, M, W, D**	Yes	Yes	Yes	Yes	Yes
Adjusted volatility	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Risk-free interest rate	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Findings (contd.)

Regression analysis for pricing errors for ITM call options

Q, M, W and D stand for portfolio rebalancing frequency of quarterly, monthly, weekly and daily, respectively. Yes indicates the pricing errors exhibit pricing bias and No indicates the pricing errors exhibit no pricing bias.

Pricing bias	**BSM**			**Leland (1985)**			**Leland (2007) cash**			**Leland (2007) stock**		
	Short	Med	Long	Short	Med	Long	Short	Med	Long	Short	Med	Long
Moneyness	Yes	Yes	Yes	Yes	**No for Q, M, W, D**	**No for D**	Yes	**No for Q, M, W, D**	**No for D**	Yes	**No for Q, M, W, D**	**No for D**
Time to maturity	Yes	Yes	Yes	Yes	Yes	**No for D**	Yes	Yes	**No for D**	Yes	Yes	**No for D**
Adjusted volatility	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Risk-free interest rate	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Conclusions

- Leland models appear to perform well in pricing call options compared to BSM model except for short-term OTM call options

OTM:

For medium-term OTM, only Leland model (2007) stock model with quarterly, monthly and weekly rebalancing prices call options well

For long-term OTM, Leland (2007) stock model prices call options - most accurate regardless of rebalancing interval

ATM:

Leland (2007) cash model – most accurate pricing model across maturities

For medium-term and long-term ATM, Leland (2007) stock and Leland (2007) cash models – approx similar pricing performance

ITM:

Leland (2007) cash model – most accurate pricing model across maturities

Conclusions (contd.)

- As rebalance frequency increases, the pricing errors of Leland models decrease for OTM, ATM and ITM call options except for short-term and medium-term OTM call options
- Prices generated from Leland models are subject to fewer and weaker pricing biases than are the prices from the BSM model
- Although the volatility and risk-free interest rate biases exist for all Leland models, it appears that Leland models are insensitive to moneyness and time to maturity biases.

Conclusions (contd.)

ATM:

Leland (2007) cash model is free of the moneyness and time to maturity biases for most rebalancing intervals compared to Leland (1985) and Leland (2007) stock models.

OTM:

Leland (2007) cash model is only free of the moneyness bias for weekly rebalancing for long-term call options and free of the time to maturity bias for weekly and daily rebalancing for medium-term call options.

ITM:

All Leland models equally free of the moneyness bias for all rebalancing for medium-term call options and only free of the time to maturity bias for daily rebalancing for long-term call options.

Conclusions (contd.)

- Advantage of Leland models over BSM model:

  Leland models able to eliminate some of the pricing biases of the BSM model.

- Results reveal that the use of adjusted volatility and the incorporation of initial costs of trading into option pricing model significantly improve option pricing effectiveness.