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RESEARCH ARTICLE

Mathematical Modelling Approach in Predicting New Mother Sea Turtle Nesting Patterns at Chagar Hutang Turtle Sanctuary, Redang Island, Malaysia

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Abstract Sea turtles, ancient marine reptiles that have survived for over 210 million years, now face unprecedented threats from human activities and climate change. This study employs mathematical modeling to predict and understand sea turtle nesting patterns at Chagar Hutang Turtle Sanctuary, Redang Island, Malaysia. We analyzed historical nesting data from 1993 to 2022 using three continuous time models: exponential growth, logistic growth, and Gompertz growth. These models were fitted to the data using Maple Software, followed by rigorous error analysis. The Gompertz model emerged as the best fit, with sum of error of 20.7, significantly outperforming the logistic (28.5) and exponential (1227.2) models. This suggests that sea turtle population growth in the area follows a sigmoidal pattern with asymmetric growth rates. The model predicts a continued increase in new mother sea turtles up to 2030, but with a decreasing growth rate, indicating the population may be approaching carrying capacity. These findings provide valuable insights for conservation planning, highlighting the need for adaptive management strategies and expanded protection efforts. Our study underscores the efficacy of mathematical modeling in predicting sea turtle population dynamics and informs evidence-based conservation strategies for these iconic marine species.

Keywords: Mother sea turtle, mathematical modelling, exponential, logistic, Gompertz model.

Introduction

Sea turtles are ancient marine reptiles that have survived for millions of years, playing a crucial role in maintaining the health of marine ecosystems. However, in recent decades, their populations have been increasingly threatened by human activities and environmental changes. Among the key nesting sites in Malaysia, Chagar Hutang Sanctuary on Redang Island Island hosts approximately one-fifth (1/5) of the total sea turtle nesting in Terengganu state, which records the highest number of nesting turtles in Peninsular Malaysia, is one of the most important habitats for the green turtle *Chelonia mydas*. This sanctuary is a critical conservation area, as it supports a significant number of nesting turtles and has been a focus of long-term research and protection efforts [1].

Understanding the nesting patterns of mother sea turtles in Chagar Hutang is essential for effective conservation planning. Nesting patterns can be influenced by environmental factors such as beach erosion, climate change, and ocean currents, as well as human-induced threats like poaching and habitat disturbance. While Chagar Hutang is a protected area, external factors such as rising sand temperatures, marine pollution, and coastal development can still impact nesting success. For instance, increasing temperatures due to climate change can lead to skewed hatchling sex ratios, ultimately affecting future population stability. Furthermore, marine debris and ghost nets pose additional risks to nesting turtles and hatchlings, reducing survival rates [2].

Figure 1 shows the long-term monitoring data indicate fluctuations in the number of nesting mother sea turtles at Chagar Hutang. Between 1993 and 2022, the population has shown varying trends, with some years experiencing a decline in nesting numbers. Factors such as natural population cycles, environmental stressors, and predation may contribute to these fluctuations. While conservation efforts have led to improvements in nesting numbers in recent years, there are concerns that future challenges, including climate change and habitat loss, may hinder population growth.

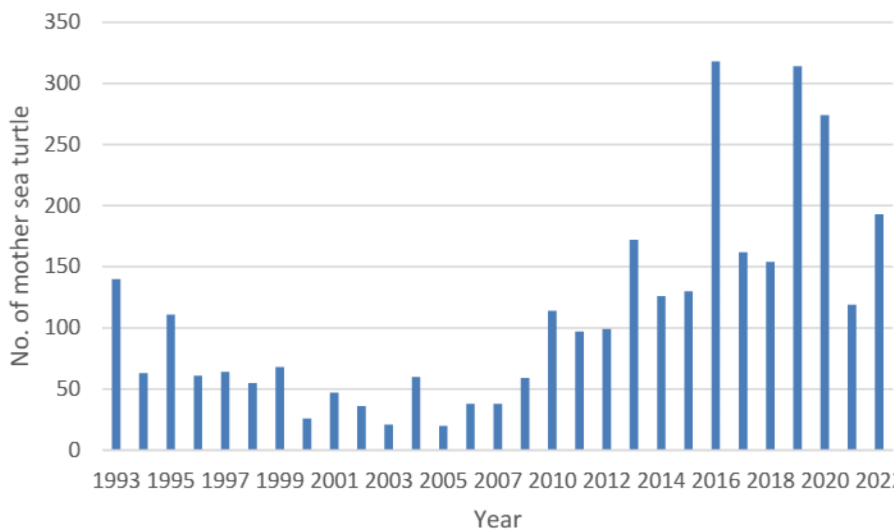


Figure 1. Number of mother sea turtles recorded at Chagar Hutang Turtle Sanctuary

To address these concerns, mathematical modelling provides a valuable tool for predicting sea turtle nesting trends and assessing population sustainability. This study employs three continuous-time models: exponential, logistic, and Gompertz growth models in order to analyze nesting patterns in Chagar Hutang Sanctuary. By fitting these models to historical nesting data, we aim to identify the best approach for predicting future trends and informing conservation strategies. Our research is motivated by [3] in which they implemented all these three models to investigate the pattern of tumor growth in human body. The findings will contribute to the development of adaptive management plans, ensuring the long-term survival of sea turtles in this vital habitat.

Methodology

Data Collection

This study was conducted at the Chagar Hutang Turtle Sanctuary (CHTS), Redang Island, focusing on sea turtle nesting behaviour. Table 1 shows the historical data spanning from 1993 to 2022, documenting the number of mother sea turtles observed on Chagar Hutang beach. The sample size was determined based on the number of nests, with maternal age differentiation achieved by categorizing mothers into old and new based on tagging on their left and right flippers.

Table 1. Historical Records of Mother Sea Turtle Observations (1993-2022)

Year	Actual Number of Mother Sea Turtle
1993	140
1994	63
1995	111
1996	61
1997	64
1998	55
1999	68
2000	26
2001	47
2002	36
2003	21
2004	60
2005	20
2006	38
2007	38
2009	59
2010	114
2011	97
2012	99
2013	172
2014	126
2015	130
2016	318
2017	162
2018	154
2019	314
2020	274
2021	119
2022	193

Mathematical Models

Three continuous time models were selected for this study: the exponential growth model, the logistic growth model, and the Gompertz growth model. These models were chosen based on their prevalence in ecological studies and their potential to capture different aspects of population growth dynamics. All the models considered here are in forms of ordinary differential equations (ODEs) and mainly referred to [4].

The exponential growth model, which describes a situation where the growth rate of a population is proportional to its current size, has been used in the early stages of population growth studies. In fact, exponential growth is usually used to study bacterial growth, due to its rapid changes with respect to time (see for examples in [5-7]). The exponential growth model is represented by the following ordinary differential equation (ODE),

$$\frac{dN}{dt} = rN, \tag{1}$$

where N representing the quantity at time t, t as the time variable, and r as the growth rate. The solution for this model is given as:

$$N(t) = N_0 e^{rt}, \tag{2}$$

where N_0 denotes the initial population.

The logistic growth model, characterized by its S-shaped curve, has been extensively applied in population dynamics and epidemiology. For instance, [8] used a stochastic simulation model based on the logistic equation to predict the long-term viability of loggerhead sea turtle populations in the Mediterranean. Their study highlighted the model's utility in assessing the impact of conservation measures on population trends. The logistic model is typically represented by the following ODE:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right), \tag{3}$$

where K denotes the carrying capacity of the environment. The solution for this model is given as:

$$N(t) = \frac{K}{1 + \left(\frac{K}{N_0} - 1\right)e^{-rt}}. \tag{4}$$

Similar to logistic model in (2), N_0 denotes the initial population.

The Gompertz model, known for its sigmoidal shape, has found applications in various biological contexts. In a study [9], a variation of the Gompertz model was used to describe the growth dynamics of green sea turtles in the Hawaiian Archipelago. Their research demonstrated the model's effectiveness in capturing the growth patterns of sea turtles over time. The Gompertz Growth Model is a sigmoid function often used to describe growth processes. The model is typically represented by the following ODE,

$$\frac{dN}{dt} = rN\ln\left(\frac{K}{N}\right). \tag{5}$$

The expression $\ln\left(\frac{K}{N}\right)$ modulates the growth rate, causing it to slow down as N approaches K. The solution for this model is given as,

$$N(t) = Ke^{-e^{-rt}}. \tag{6}$$

In this study, we will use the solutions (2), (4) and (6) of the three models to fit with the time series data for mother sea turtle population and later the best model will be used for prediction.

Parameterization for Growth Rate

The analysis was conducted in Rstudio using the following libraries such as readxl for data import, deSolve for solving ordinary differential equations, tidyverse and purrr for data manipulation, and ggplot2 for visualization.

```
library(readxl)
library(deSolve)
library(tidyverse)
library(purrr)
library(ggplot2)
```

Then a parameter grid was generated to cover growth rates from 0.1 to 10.0 in increments of 0.001, with a constant carrying capacity (K) of 100. This carrying capacity value is chosen by taking the average of all the number of mother sea turtles in Table 1. The code for generating the grid is as follows:

```
r <- seq(0.1, 10.00, by = 0.001)
K <- 100
param_grid <- expand.grid(r = r, K = K)
```

All the simulations were conducted using three different growth models, with the corresponding equations:

```
GompertzMod <- function(Time, State, Pars) {
    with(as.list(c(State, Pars)), {
        dN <- r * N * log(K / N)  # Gompertz model
        return(list(c(dN)))
    })
}

LogisticMod <- function(Time, State, Pars) {
    with(as.list(c(State, Pars)), {
        dN <- r * N * (1 - N / K)  # Logistic model
        return(list(c(dN)))
    })
}

ExponentialMod <- function(Time, State, Pars) {
    with(as.list(c(State, Pars)), {
        dN <- r * N  # Exponential model
        return(list(c(dN)))
    })
}
```

Error analysis was performed by comparing predicted populations against actual data to identify optimal parameter values, implemented through the (calculate_errors) function. The results of the simulations provided forecasts of the sea turtle population from 1993 to 2022, allowing for an assessment of trends and potential conservation strategies.

```
Predicted_Turtle <- map(1:nrow(param_grid),
                        ~ simulate(param_grid$r[.x],
                                   param_grid$K[.x])) %>%
    Filter(function(x) nrow(x) == 30, .)
```

To choose the best model for prediction, the error analysis is performed to compare actual data with predicted model data. This absolute error analysis is widely used by many researchers including by Dahri *et al.* [10]. The three models will be compared in term of error percentage and the best fitted model is chosen based on the smallest error given by the model. The formula of error is given as follows:

$$\text{Error} = \left|\frac{\text{Actual Value} - \text{Estimated Value}}{\text{Actual Value}}\right|. \tag{7}$$

Results and Discussion

Simulation Results for the Three Models

This section compares three population growth models, which are exponential growth, logistic growth, and Gompertz model to analyze the nesting patterns of new mother sea turtles. The initial value used is based on the initial data of number of mother sea turtles in the year of 1993. in Table 1 which is 140. The values of parameters used are: $r = 0.18$ and $K = 100$. The results of our analysis revealed significant differences in the predictive power of these models. Table 2 presents a comparison between the actual number of new mother sea turtles observed each year and the predictions made by each of the three models. This table provides a year-by-year breakdown, allowing for a detailed comparison of model performance. Notably, the Gompertz model's predictions appear to track the actual data more closely than the other models, especially in later years.

Table 2. Simulation results for all the three models

Year	Exponential Growth	Logistic Growth	Gompertz Growth
1993	140.0	140.0	36.8
1994	167.6	131.3	43.4
1995	200.7	124.9	49.8
1996	240.2	120.0	55.8
1997	287.6	116.2	61.5
1998	344.3	113.1	66.6
1999	412.3	110.7	71.2
2000	493.6	108.8	75.3
2001	590.9	107.3	78.9
2002	707.4	106.0	82.0
2003	847.0	105.0	84.8
2004	1014.0	104.1	87.1
2005	1214.0	103.4	89.1
2006	1453.4	102.8	90.8
2007	1740.0	102.4	92.3
2009	2083.2	102.0	93.5
2010	2494.0	101.6	94.5
2011	2985.9	101.4	95.4
2012	3574.7	101.1	96.2
2013	4279.7	100.9	96.8
2014	5123.8	100.8	97.3
2015	6134.2	100.7	97.7
2016	7344.0	100.5	98.1
2017	8792.4	100.5	98.4
2018	10526.4	100.4	98.7
2019	10526.4	100.3	98.9
2020	15087.8	100.3	99.1
2021	18063.4	100.2	99.2
2022	21625.8	100.2	99.4

Model Performance Comparison

To quantify the performance of each model, we conducted an error analysis by calculating the sum of errors, obtained by comparing the predicted data with actual data (as shown in Table 1). The results are presented in Table 3.

Table 3. Error Analysis for each model

Model	Sum of Error
Exponential	1227.19945
Logistic	28. 50142
Gompertz	20.72965

Table 3 summarizes the total error for each model, calculated using the formula described in the methodology. This table provides a simple, quantitative comparison of model performance. The Gompertz model's substantially lower error (20.7) compared to the exponential (1227.2) and logistic (28.5) models, which numerically confirms that Gompertz model's is the best-fitted model for this dataset. These results clearly indicate that the Gompertz model significantly outperformed the other two models in predicting new mother sea turtle nesting patterns.

Model Performance and Ecological Implications

In this section, we show the comparison between the three models with actual data graphically. All the graphs are plotted using Microsoft Excel. Figure 2 compares the exponential growth model with actual data on the number of mother sea turtles from 1993 to 2022. The actual data, represented by a dotted line, shows relatively stable numbers with minor fluctuations, generally staying under 100. In contrast, the exponential growth model, shown as a dashed-dotted line, predicts a steep and continuous increase, especially after 2010, resulting in a very high number of mother sea turtles by 2022. This model does not align with the actual data, suggesting an unrealistic and overly optimistic increase.

Meanwhile, Figure 3 compares the logistic growth model with actual data. The actual data, again represented by a dotted line, shows considerable fluctuations over time, with peaks and troughs, indicating variability in the population. The logistic growth model, shown as a dashed line, assumes a carrying capacity $K = 100$ and shows a slight decline initially before stabilizing around this capacity. However, the actual data does not align well with this model either, as the numbers exceed the carrying capacity and show higher fluctuations.

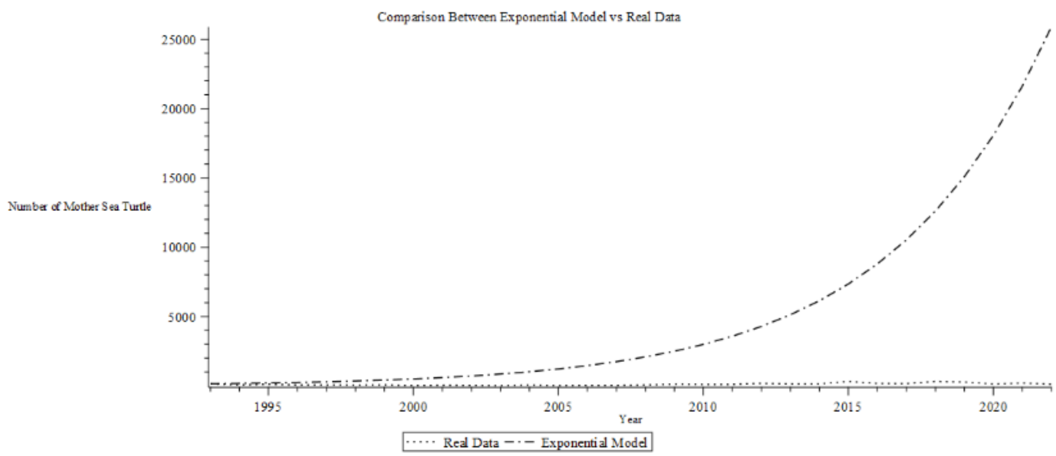


Figure 2. Exponential model fitted on actual data points

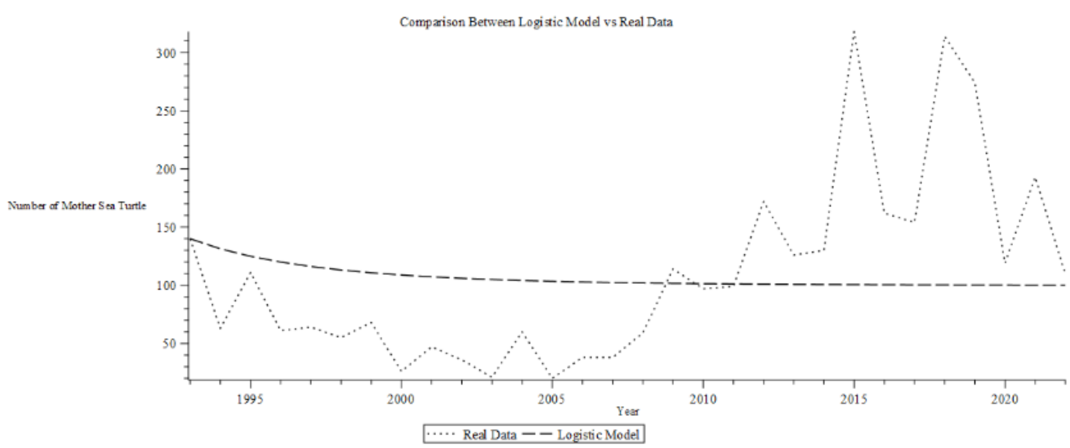


Figure 3. Logistic model fitted on actual data points

Moving on to Figure 4, the chart compares the actual number of mother sea turtles with the number predicted by the Gompertz model from 1993 to 2022. The actual data, represented by a dotted line, shows significant fluctuations over the years, with some periods displaying much higher numbers of mother sea turtles, such as between 2010 and 2015 and again around 2020. This variability is marked by several peaks and troughs. In contrast, the Gompertz model, depicted by the solid line, predicts a smoother, more consistent trend. Initially, the model forecasts a gradual increase in the number of mother sea turtles from 1993 to around 2005, followed by a plateau where the numbers stabilize with little to no increase until 2022.

When comparing the two, it is evident that the actual data exhibits much more variability than the predicted data from the Gompertz model. The model accurately captures the initial increasing trend from 1993 to about 2005 but fails to predict the subsequent fluctuations and peaks observed in the actual data. Specifically, the model's prediction of stabilization post-2005 does not align with the actual data, which continues to show significant year-to-year variation, and it underestimates the number of mother sea turtles during peak years, such as around 2015 and 2020.

In comparison, both exponential and logistic models fail to accurately predict the actual population trends of mother sea turtles. The exponential growth model assumes unlimited resources and no constraints, leading to unrealistic predictions. The logistic growth model considers a carrying capacity, which is more realistic, but the chosen capacity does not match the actual population dynamics. When comparing the two models with Gompertz model, this suggests that Gompertz gives a closer prediction even though it does not accurately predict the large fluctuations seen in the actual data.

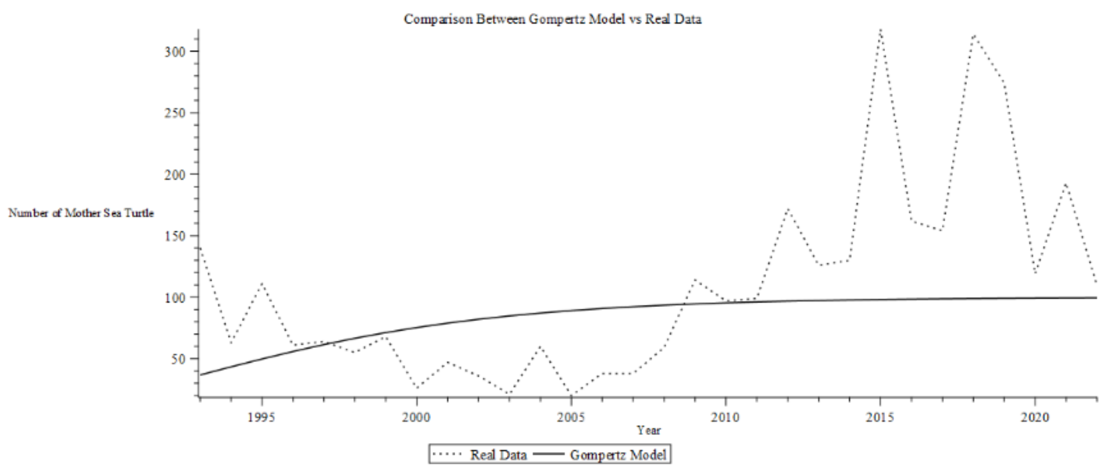


Figure 4. Gompertz model fitted on actual data points

Moreover, the Gompertz model's characteristic with asymmetric growth rates aligns well with the biological realities of sea turtle populations:

1. Initial Slow Growth Phase: This phase, evident in the early years of Figure 4, likely represents the time required for sea turtles to reach sexual maturity, typically 20-30 years for most species. The slow initial growth in the model may also reflect the challenges faced by young adults in their first nesting attempts. Factors such as inexperience in nest site selection, vulnerability to predators, and sensitivity to environmental disturbances could contribute to this slow start.
2. Rapid Growth Phase: The period of accelerated growth in the model, visible in the middle years of Figure 4, could indicate successful conservation efforts taking effect. This might include increased protection of nesting beaches, reduced bycatch in fisheries, or improvements in ocean ecosystem health. It's also possible that this phase represents a period of particularly favourable environmental conditions, such as optimal sea temperatures or abundant food resources.
3. Decelerating Growth Phase: As the model predicts a slowing growth rate in later years, shown in the flattening curve of Figure 4, this suggests the population may be approaching the carrying capacity of the Chagar Hutang nesting area. This deceleration could be due to density-dependent factors such as competition for optimal nesting sites or food resources.

Future Predictions and Implications for Conservation

Figure 5 extends the Gompertz model's predictions to 2030. The x-axis shows the years from 1993 to 2030, with the period from 2023 to 2030 being the future prediction. The y-axis represents the number of new mother sea turtles. The graph likely shows the actual data up to 2022 and then continues with the model's predictions up to 2030. This figure illustrates the expected continued growth but at a decreasing rate, as evidenced by the flattening of the curve towards 2030. This projection has important implications for conservation efforts:

1. Adaptive Management: As the population approaches carrying capacity, management strategies may need to shift from focusing solely on population growth to maintaining population stability and genetic diversity.
2. Habitat Protection and Expansion: The predicted growth underscores the need not only to protect current nesting beaches but also to identify and conserve potential new nesting sites that could support the expanding population.
3. Climate Change Preparedness: With the model predicting population trends up to 2030, it's crucial to consider the potential impacts of climate change. Rising sea levels and increasing temperatures could affect the availability and quality of nesting habitats.
4. Integrated Ecosystem Management: The success of sea turtle conservation is intrinsically linked to the health of marine ecosystems. Management plans should adopt a holistic approach, considering factors like fisheries management, marine protected areas, and pollution control.

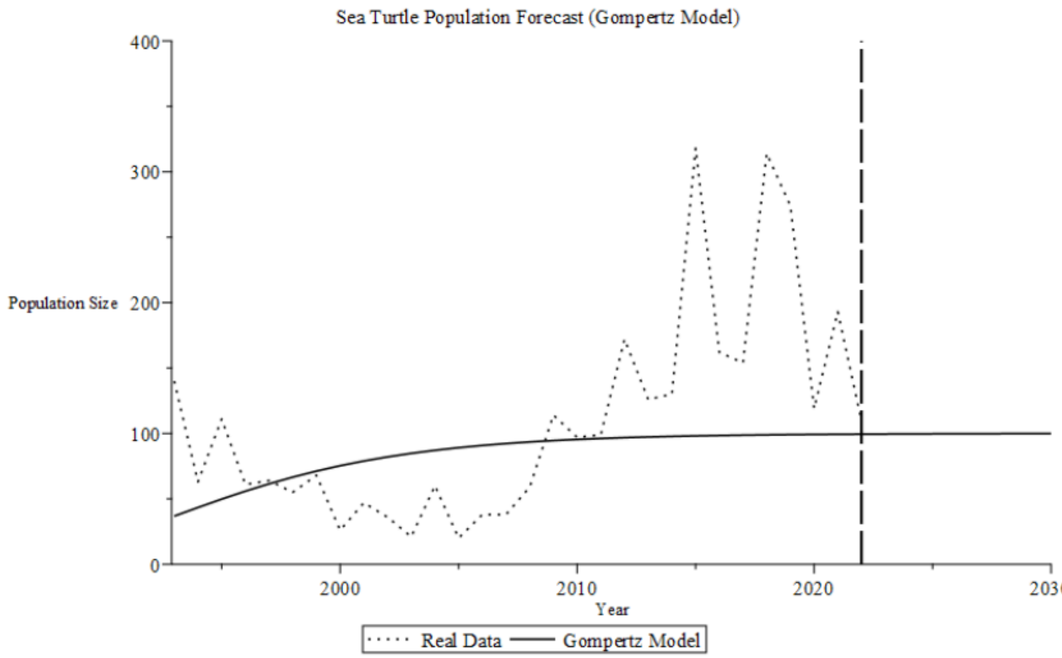


Figure 5. Gompertz model predictions extended to 2030

Parameterization of the Growth Rate (r)

A critical aspect of the modeling process involves exploring a wide range of growth rates (r), spanning from 0.1 to 10.0 with increments of 0.001, while maintaining the carrying capacity (K) constant at 100. This extensive parameter grid facilitates a thorough exploration of how varying growth rates influence population dynamics under a fixed environmental constraint (K). The use of the (***expand.grid***) function efficiently generates all possible combinations of these parameters, setting the stage for robust simulations and ensuring a comprehensive examination of potential growth scenarios (Figure 6).

The parametric grid, which includes 9,901 combinations of growth rates and a fixed carrying capacity, allows for a detailed simulation of the sea turtle population dynamics. This approach provides valuable insights into how variations in the growth rate influence the predicted population over time. By examining such a wide range of possible growth rates, the model can capture different possible growth scenarios and predict how the population might evolve under varying conditions.

param_grid	9901 obs. of 2 variables

	r	K
1	0.100	100
2	0.101	100
3	0.102	100
4	0.103	100
5	0.104	100
6	0.105	100
7	0.106	100
8	0.107	100
9	0.108	100
10	0.109	100
11	0.110	100
12	0.111	100
13	0.112	100
14	0.113	100
15	0.114	100

Showing 1 to 15 of 9,901 entries, 2 total columns

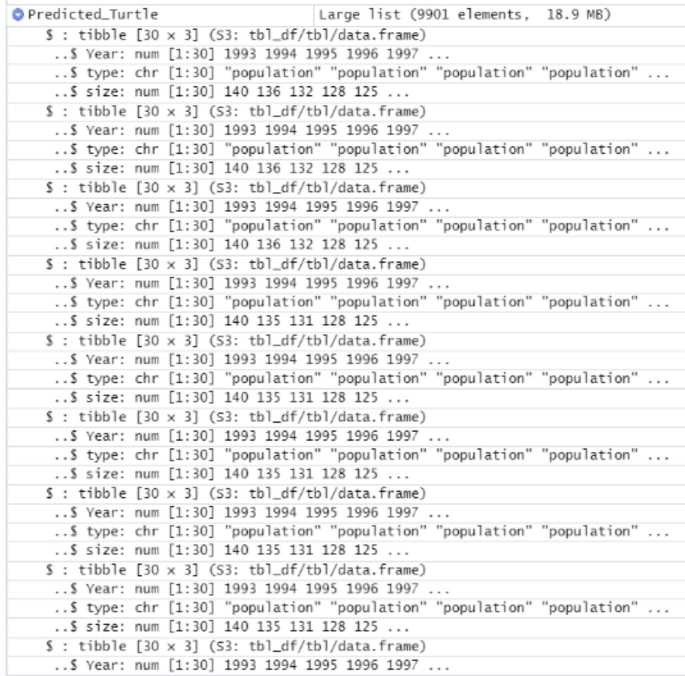
Figure 6. Parametric grid with 9,901 combinations of growth rate (r) values from 0.1 to 10.0, while maintaining the carrying capacity (K) constant at 100. This grid allows for a comprehensive exploration of the population dynamics

For each parameter combination in the grid, simulations are conducted by initializing the sea turtle population at 140 individuals and projecting population changes from 1993 to 2022. This is achieved using the ordinary differential equation (ODE) solver from the ***deSolve*** package, specifically the lsoda method, which is capable of handling potential stiffness in the equations with a maximum of 5000 steps. The use of this solver ensures the robustness of the simulations, allowing the model to accommodate a wide range of growth scenarios. The simulation results are then organized into a tidy format, facilitating subsequent analysis and visualization.

Simulations across all 9,901 parameter combinations (r and K) are aggregated into a unified dataset, ensuring that only simulations spanning the full 30-year period are retained. This filtering step is crucial to maintain consistency when comparing the predicted population data with the actual observed data from 1993 to 2022. The alignment of predicted data with the real timeframe ensures that the analysis remains focused and relevant to the study period.

The sea turtle population predictions were generated for each combination of growth rate and carrying capacity. The predictions align closely with the observed data, showcasing the model's accuracy in capturing population trends over the years. This demonstrates the ability to adapt to various growth scenarios and fine-tune the parameters to optimize prediction accuracy (Figure 7). The parametric grid thus provides a robust approach, ensuring that the model explores a wide spectrum of potential growth trajectories and predicts population dynamics effectively.

```
Predicted_Turtle          Large list (9901 elements,  18.9 MB)
```



```
Predicted_Turtle                Large list (9901 elements,  18.9 MB)
  $ : tibble [30 × 3] (S3: tbl_df/tbl/data.frame)
  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
  ..$ type: chr [1:30] "population" "population" "population" "population" ...
  ..$ size: num [1:30] 140 136 132 128 125 ...
  $ : tibble [30 × 3] (S3: tbl_df/tbl/data.frame)
  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
  ..$ type: chr [1:30] "population" "population" "population" "population" ...
  ..$ size: num [1:30] 140 136 132 128 125 ...
  $ : tibble [30 × 3] (S3: tbl_df/tbl/data.frame)
  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
  ..$ type: chr [1:30] "population" "population" "population" "population" ...
  ..$ size: num [1:30] 140 136 132 128 125 ...
  $ : tibble [30 × 3] (S3: tbl_df/tbl/data.frame)
  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
  ..$ type: chr [1:30] "population" "population" "population" "population" ...
  ..$ size: num [1:30] 140 135 131 128 125 ...
  $ : tibble [30 × 3] (S3: tbl_df/tbl/data.frame)
  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
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  ..$ size: num [1:30] 140 135 131 128 125 ...
  $ : tibble [30 × 3] (S3: tbl_df/tbl/data.frame)
  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
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  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
  ..$ type: chr [1:30] "population" "population" "population" "population" ...
  ..$ size: num [1:30] 140 135 131 128 125 ...
  $ : tibble [30 × 3] (S3: tbl_df/tbl/data.frame)
  ..$ Year: num [1:30] 1993 1994 1995 1996 1997 ...
```
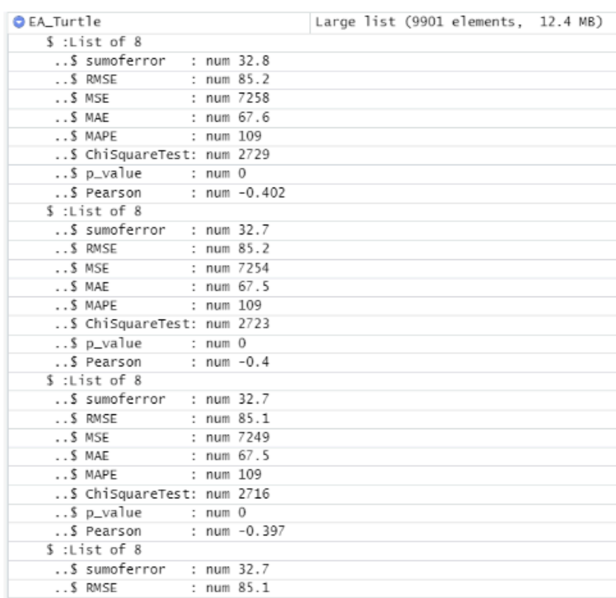

Figure 7. Predicted sea turtle population based on the parametric grid. The figure illustrates how different growth rates influence the predicted population, showing alignment with real-world data

A crucial part of the analysis was the error assessment between the predicted population and the actual population data. By comparing the model output with real data, it was possible to evaluate the performance of each parameter set, leading to the identification of parameter values that minimized prediction error. Figure 8 illustrates the error analysis across different parameter sets, showing the deviation between the predicted and actual population values. The error analysis highlights the model's strong predictive power when the appropriate parameters are selected.



```
EA_Turtle                       Large list (9901 elements,  12.4 MB)
  $ :List of 8
  ..$ sumoferror    : num 32.8
  ..$ RMSE          : num 85.2
  ..$ MSE           : num 7258
  ..$ MAE           : num 67.6
  ..$ MAPE          : num 109
  ..$ ChiSquareTest: num 2729
  ..$ p_value       : num 0
  ..$ Pearson       : num -0.402
  $ :List of 8
  ..$ sumoferror    : num 32.7
  ..$ RMSE          : num 85.2
  ..$ MSE           : num 7254
  ..$ MAE           : num 67.5
  ..$ MAPE          : num 109
  ..$ ChiSquareTest: num 2723
  ..$ p_value       : num 0
  ..$ Pearson       : num -0.4
  $ :List of 8
  ..$ sumoferror    : num 32.7
  ..$ RMSE          : num 85.1
  ..$ MSE           : num 7249
  ..$ MAE           : num 67.5
  ..$ MAPE          : num 109
  ..$ ChiSquareTest: num 2716
  ..$ p_value       : num 0
  ..$ Pearson       : num -0.397
  $ :List of 8
  ..$ sumoferror    : num 32.7
  ..$ RMSE          : num 85.1
  ..$ MSE           : num 7245
```

Figure 8. Error analysis of predicted sea turtle population compared to real data across varying parameter values

To further enhance our understanding of model performance, we conducted a summary of the error analysis from the 9,901 simulations. Figure 9 presents a concise overview of the minimum and maximum values for each error metric calculated during the analysis. This summary allows us to identify which parameters yielded the best predictions by providing a clear visual representation of the range of errors associated with different parameter combinations.

	Metric	Value
1	Maximum SUMERROR	32.78608014
2	Minimum SUMERROR	27.09278076
3	Maximum RMSE	85.19425460
4	Minimum RMSE	80.21702094
5	Maximum MSE	7258.06101748
6	Minimum MSE	6434.77044816
7	Maximum MAE	67.55231820
8	Minimum MAE	59.76671783
9	Maximum MAPE	109.28693380
10	Minimum MAPE	90.30926918
11	Maximum ChiSquareTest	2729.49813898
12	Minimum ChiSquareTest	1999.35274028
13	Maximum Chi-Square p-value	0.00000000
14	Minimum Chi-Square p-value	0.00000000
15	Maximum Pearson	0.07554496
16	Minimun Pearson	-0.40187013

Figure 9. Summary of error analysis for the 9,901 simulations. The figure shows the minimum and maximum values for key error metrics, aiding in the identification of the most accurate parameter sets for predicting sea turtle populations

Through this parametric approach, it is evident that small variations in the growth rate r significantly affect the population predictions, while the carrying capacity K remains relatively stable in the model. This suggests that future conservation efforts should focus on understanding the factors that influence the growth rate, as they are likely to have the greatest impact on the sea turtle population dynamics.

Conclusions

In conclusion, we have successfully compared three mathematical models for the mother sea turtles' data for year 1993 to 2022. This study demonstrates that the Gompertz growth model showed the best model in predicting new mother sea turtle nesting behaviour at Chagar Hutang Turtle Sanctuary. The model has least error as compared to exponential and logistic growth models suggests that sea turtle population growth in this area follows a pattern of initial slow growth, followed by rapid increase, and then a gradual slowing as it approaches carrying capacity.

The parameterization process, involving a comprehensive exploration of growth rates (r) and carrying capacities (K), was essential for optimizing the model's predictions. By systematically evaluating 9,901 parameter combinations, the study was able to identify the most effective growth scenarios for this population. This extensive parameter grid not only enhanced the robustness of the model but also provided crucial insights into how varying growth rates influence population dynamics.

These findings provide valuable insights for conservation planning and management of sea turtle populations. The predicted continued growth, albeit at a decreasing rate, suggests that current conservation efforts may be effective but may need to be adapted to accommodate a larger nesting population in the future. However, the limitations of the model, including its inability to capture short-term fluctuations and the still significant error rate, highlight the need for continued research and more complex modelling approaches. Future studies should aim to incorporate additional environmental and anthropogenic factors to improve predictive accuracy.

By integrating this model-based approach with ongoing ecological research and conservation efforts, we can develop more effective strategies for protecting sea turtle populations in the face of ongoing environmental changes and human pressures. This study contributes to the growing body of research using mathematical modelling in ecology and conservation biology, demonstrating its potential in informing evidence-based conservation strategies for these endangered marine species.

Conflicts of Interest

The author(s) declare(s) that there is no conflict of interest regarding the publication of this paper.

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