Fuzzy TOPSIS Method in the Selection of Investment Boards by Incorporating Operational Risks

Elissa Nadia Madi, and Abu Osman Md Tap

Abstract— **Multi Criteria Decision Making (MCDM) involves not only attributes that are precise or crisp, but also values that are not deterministic. Currently, Fuzzy TOPSIS presents a solution for decision makers when dealing with real world data that are usually multi attributes and involves a complex decision making process. In this work, an application of this method is demonstrated in the selection of Investment Boards by taking into account the operational risks involved.**

Index Terms— **Fuzzy TOPSIS, Multi Criteria Decision Making (MCDM), Operational Risks**

I. INTRODUCTION

Nowadays, dynamics and risky global financial environment had caused the stock investors to become more beware during investing process. Hence, investment assessment is important to immune the invested stocks from exposed by that risk.

In most of real world situations, usually decision makers are confronted with multiple criteria to be considered before any decision can be made. This is the case of Multi Criteria Decision Making (MCDM); a case with the aim to find the overall preferences among the available alternatives.

In addition, when the attributes are not deterministic; the fuzzy logic approach is usually adopted. One of the most popular methods in MCDM is the Technique for Order Preference by Similarity or TOPSIS. Hence, in the case of attributes that are not deterministic, fuzzy TOPSIS method will be used.

The theories were applied for choosing the best board in Bursa Malaysia stock trading investment. It will choose based on three criteria which are Volume Trading Stock, Market Valuation and Value of the stock.

Manuscript received January 7, 2011; revised February,9, 2011. First author, Elissa Nadia Madi currently work as a lecturer at Faculty of Informatics, University Sultan Zainal Abidin, Terengganu, Malaysia. Her major field is in Fuzzy Set Theory,(corresponding author phone: +6012-6654563, Fax: 609-6673412,e-mail: elissa@unisza.edu.my).

Abu Osman Md. Tap currently work as a Professor at Dept. of Information Systems, International Islamic University Malaysia. His major field is in Topology and Algebra. (corresponding author phone: +603-61965646, Fax: +603-61965179, email: abuosman@kict.iiu.edu.my).

Many business firms only focus on common financial risks (interest rate risk, market risk and etc.) to maintain their business to keep running. However, the question of how the firms manage their business operations is a major factor to maintain their performance [1].

In real world, dynamic and risky global financial environment had caused stock investors to be more aware of the investment process. Operational risks are one crucial factor that determines the final outcomes of an investment hence making decision making process critical in order to avoid expected and unexpected losses. As a result, managing operational risks is usually done by a firm for the purpose of adding value by reducing the risks associated with the firm's earnings [1]. That is the reason why we propose the incorporation of operational risks in our study.

The rest of this paper is organized as follows. Section 2 outlines related works on Fuzzy TOPSIS whereas Section 3 presents the preliminaries on Fuzzy TOPSIS. An example application of the model is described in Section 4, followed by the conclusion in Section 5.

II. RELATED WORKS

TOPSIS was proposed by Hwang and Yoon in 1981 [2]. In this method, the main concept is that the most preferred alternative should have the shortest distance from the Positive Ideal Solution (PIS) and the longest distance from the Negative Ideal Solution (NIS) [3]. Based on Wang and Elhag [4], PIS is the one that maximizes the benefit criteria and minimizes the cost criteria, while the NIS functions in the opposite way. As opposed to the original application of TOPSIS where the weight of the criteria and the ratings of alternatives are known precisely, many real-life decision problems are confronted with unquantifiable, incomplete and non-obtainable information [5] that make precise judgment impossible. This is when fuzzy TOPSIS comes into play where the criteria weights and alternative ratings are given by linguistic variables, expressed by fuzzy numbers.

In the year 2000, Chen [6] had used an algorithm of a group multi-criteria decision making that is composed of the following steps in Table I [6] :

TABLE 1
STEPS OUTLINING THE ALGORITHM OF A GROUP MULTI-CRITERIA DECISION MAKING

Step	Remarks
1.	Identify the evaluation criteria (Usually done by a committee of decision-makers)
2.	Choose appropriate linguistic variables (based on the importance weight of the criteria) and the linguistic ratings for alternatives with respect to the criteria
3.	Aggregate the weight of the criteria to get the aggregated fuzzy weight $\hat{w}_j$ of criterion C_j and pool the decision makers' opinions to get the aggregated fuzzy rating x_{ij} of alternative A_i under criterion C_j.
4.	Construct the fuzzy decision matrix and the normalized fuzzy decision matrix
5.	Construct the weighted normalized fuzzy decision matrix
6.	Determine the FPIS and NPIS
7.	Calculate the distance of each alternative from FPIS and NPIS, respectively
8.	Calculate the closeness coefficient of each alternative
9.	Determine the ranking order of all alternatives according to the closeness coefficients.

III. PRELIMINARIES

This section briefly outlines some basic definitions of fuzzy sets from [7 – 10]-:

Definition 3.1. A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}$ (x)which associates with each element x in X a real number in the interval [0,1].The function value $\mu_{\tilde{A}}$(x) is termed the grade of membership of x in $\tilde{A}$ [7].

Definition 3.2. A triangular fuzzy number n can be defined by a triplet (n_1, n_2, n_3) shown in Fig. 1. The membership function $\mu_{\tilde{n}}(x)$ is defined as [8] :

$$\mu_n(x) = \begin{cases} 0, & x < n_1, \\ \frac{x-n_1}{n_2-n_1} & n_1 \le x \le n_2, \\ \frac{x-n_3}{n_2-n_3} & n_2 \le x \le n_3, \\ 0, & x > n_3. \end{cases} \qquad (1)$$

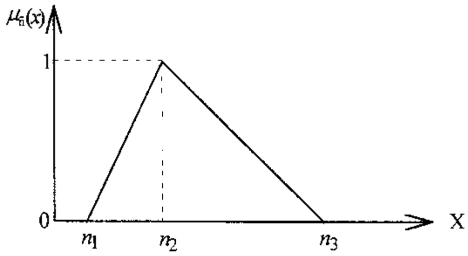

Fig. 1. A triangular fuzzy number n

Definition 3.3. Let m =(m_1,m_2,m_3) and n =(n_1,n_2,n_3) be two triangular fuzzy numbers. If m = n, then m_1=n_1, m_2=n_2 and m_3=n_3.

Definition 3.4. D is called a fuzzy matrix, if at least an entry in D is a fuzzy number [9].

Definition 3.5. A linguistic variable is a variable whose values are linguistic terms [10]. The concept of linguistic variable is very useful in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions [9]. For example, "weight" is a linguistic variable and its values are very low, low, medium, high, very high, etc. These linguistic values can also be represented by fuzzy numbers.

IV. THE PROPOSED METHOD

In this study, TOPSIS method is used in the determination of final ranking from a group of investment boards. The method is calculated as follows:

Let MCDM problem has n alternatives A_1, A_2,...,A_n, and m criteria, C_1, C_2,...,C_m. Each alternative will take a consideration with respect to criterion m . The ratings of criteria can be concisely expressed in matrix format as $X = (x_{ij})_{n\times m}$ and $W = (w_1, \dots, w_m)$, where x_{ij} (i = 1,...,; j = 1,...,n) and w_j (j = 1,...,n) are the fuzzy rating of alternative A_i (i = 1,...,m) with respect to criterion C_j (j = 1,...,m) and the weight of criterion C_j (j = 1,...,m), respectively. The method is calculated using the following steps :

(a) Decision matrix, $X = (x_{ij})_{n\times m}$ is normalized via Eq. (2):

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{n} x_{kj}^2}}, \; i = 1, \dots, n; \; j = 1, \dots, m \qquad (2)$$

Weighted normalized decision matrix is formed:

$$V = (v_{ij})_{n\times m} :$$

$$v_{ij} = w_j r_{ij}, \; i = 1, \dots, n; \; j = 1, \dots, m \qquad (3)$$

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(b) Positive ideal solution (PIS) and negative ideal solution (NIS) are determined:

$$A^* = \{v_1^*, \dots, v_m^*\} = \{(\max_j v_{ij} \,|j \in \varphi_b), (\min_j v_{ij} \,|j \in \varphi_c)\} \quad (4)$$

$$A^- = \{v_1^-, \dots, v_m^-\} = \{(\min_j v_{ij} \,|j \in \varphi_b), (\max_j v_{ij} \,|j \in \varphi_c)\} \quad (5)$$

(c) The distance of each alternative from PIS and NIS are calculated using Euclidean distance formula: :

$$D_i^* = \sqrt{\sum_{j=1}^m \left(v_{ij} - v_j^*\right)^2,\ i = 1, \dots, n} \quad (6)$$

$$D_i^- = \sqrt{\sum_{j=1}^m \left(v_{ij} - v_j^-\right)^2,\ i = 1, \dots, n} \quad (7)$$

(d) The closeness coefficient of each alternative is calculated:

$$CC_i = \frac{d_i}{d_i^* + d_i}, \ i = 1, 2,..m. \quad (8)$$

(e) By comparing CC_i values, the ranking of alternatives are determined.

V. Application of Fuzzy TOPSIS in the Selection of Investment Boards on *Bursa Malaysia*

Investors may want to evaluate the performance of the stocks to include in their portfolio. In this case, the first step is they have to choose which boards they want to invest with respect to operational risk which might be exist in each of the stocks. King [1] states that there are three criteria that will provide good insights in the evaluation of stock performance with respect to operational risks. The three criteria are Market Valuation, Stock Trading Volume and Stock Trading Value.

The data used here were gathered from the Bursa Malaysia website starting from January 2005 until December 2006. In this study, the scale of the importance of various criteria and scale of the priorities were expressed in the form of linguistic variables. Linguistic variables are presented as triangular fuzzy number as in Tables II and III. Level of importance of each criterion can be obtained directly or indirectly using paired comparisons. In this study, it is proposed that the decision-makers use the linguistic variables (see Table II and III) to assess the importance of each criteria and the alternative priorities for the criteria.

TABLE II
LINGUISTIC VARIABLES FOR THE IMPORTANCE WEIGHT OF EACH CRITERION

Very Not Important (VNI)	*(0, 0, 0.1)*
Not Important (NI)	*(0, 0.1, 0.3)*
Somewhat Not Important (SNI)	*(0.1, 0.3, 0.5)*
Medium (M)	*(0.3, 0.5, 0.7)*
Somewhat Important (SI)	*(0.5, 0.7, 0.9)*
Important (I)	*(0.7. 0.9, 1.0)*
Very Important (VI)	*(0.9, 1.0, 1.0)*

TABLE III
LINGUISTIC VARIABLES FOR THE RATINGS

Very Not Poor (VNP)	*(0, 0, 1)*
Poor (P)	*(0, 1 .3)*
Medium Poor (MP)	*(1,3,5)*
Fair (F)	*(3, 5, 7)*
Medium Good (MG))	*(5, 7, 9)*
Good (G)	*(7. 9, 10)*
Very Good (VG)	*(9, 10,0)*

The three alternatives of Investment Boards on Bursa Malaysia are as follows :

1) The Main Board, *(A_1).*
2) The Second Board, *(A_2).*
3) The MESDAQ Market, *(A_3)*

In addition, the three criteria to be considered in this study are :

1) Market valuation (RM billion), *(C_1)*
2) Stock Trading Volume (million units), *(C_2)*
3) Stock Trading Value (RM million), *(C_3)*

The next stage involves six steps as outlined in Section V.

Step 1*:* The decision-makers use the linguistic weighting variables (See Table II) for determining the level of importance of criteria and the results are summarized in Table IV.

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TABLE IV
THE IMPORTANCE WEIGHT OF THE CRITERIA

	D_1	D_2	D_3
C_1	I	VI	M
C_2	VI	VI	VI
C_3	VI	I	M

Step 2: Decision makers use the linguistic weighting variables (See Table III) to determine the priority of each criterion and the alternative is summarized in Table V.

TABLE V
THE RATINGS OF THE THREE CANDIDATES UNDER ALL CRITERIA

Criteria	Alternative	The decision maker		
		D_1	D_2	D_3
C_1	A_1	VG	VG	MG
	A_2	G	MG	F
	A_3	G	G	G
C_2	A_1	G	VG	MG
	A_2	VG	G	VG
	A_3	MG	MG	F
C_3	A_1	G	G	MG
	A_2	G	MG	MG
	A_3	F	G	G

***Step* 3:** Changing the linguistic evaluation (shown in Table IV and V) to the triangular fuzzy numbers (Table VI) and then build a fuzzy decision matrix and determine the weight for each criterion as presented in Table VII.

TABLE VI
FUZZY DECISION MATRIX

	C_1	C_2	C_3
A_1	*(7.7, 9.0, 7.9)*	*(7, 8.7, 7.9)*	*(6.3, 8.3, 7.9)*
A_2	*(5, 7, 8.7)*	*(8.3, 9.7, 10)*	*(5.7, 7.7, 3.9)*
A_3	*(7, 9, 10)*	*(4.3, 6.3, 8.3)*	*(5.7, 7.7, 9.0)*

TABLE VII
FUZZY WEIGHT FOR ALL CRITERIA

	C_1	C_2	C_3
Weight	*(0.63, 0.80, 0.90)*	*(0.9, 1.0, 1.0)*	*(0.63, 0.80, 0.90)*

Step 4: Construct a normalized fuzzy decision matrix as shown in Table VIII. The step of data normalization is necessary to overcome differences between the units. Normalization also enables valuation measure in the same range of values which is usually between zero and one. In the range system, 1 represents the highest value in upward movement where 0 represents the lowest value.

TABLE VIII
FUZZY NORMALIZED DECISION MATRIX FOR THE SELECTION OF STOCK LISTINGS OF THE BOARDS

	C_1	C_2	C_3
A_1	*(0.8, 0.9, 1.0)*	*(0.7, 0.9, 1.0)*	*(0.7, 0.9, 1.0)*
A_2	*(0.5, 0.7, 0.9)*	*(0.8, 1.0, 1.0)*	*(0.6, 0.8, 1.0)*
A_3	*(0.7, 0.9, 1.0)*	*(0.4, 0.6, 0.8)*	*(0.6, 0.8, 0.9)*

Step 5: Construct a Weighted Normalized Fuzzy Decision Matrix as shown in Table IX. To get multi criteria index, data from each of the criteria need to be aggregated. Various methods can be done to implement them. An example of this is to use the weighted mean. There are two methods for calculating weighted mean, first is an arithmetic mean and second is by using geometric mean. Index based on arithmetic mean is generally more popular because of easily understood and implemented.

TABLE IX
WEIGHTED NORMALIZED FUZZY DECISION MATRIX FOR SELECTION OF BOARD STOCK LISTING

	C_1	C_2	C_3
A_1	*(0.5, 0.7, 0.9)*	*(0.6, 0.9, 1.0)*	*(0.4, 0.7, 0.9)*
A_2	*(0.3, 0.6, 0.8)*	*(0.8, 1.0, 1.0)*	*(0.4, 0.6, 0.9)*
A_3	*(0.4, 0.7, 0.9)*	*(0.4, 0.6, 0.8)*	*(0.4, 0.6, 0.8)*

Table IX is a weighted normalized fuzzy decision matrix, taking into account the weights as determined by decision makers. The next step is to get the Fuzzy Positive Ideal Solutions (FPIS), (A^*) and Fuzzy Negative Ideal Solutions (FNIS), (A^-).

Step 6: To assign both the ideal solutions, the method used by Chen [4] is adopted as it can easily be understood. Consequently, the FPIS (A^*) and FNIS (A^-) are defined as the following :

$$A^* = (1, 1, 1) \text{ and } A^- = (0, 0, 0).$$

After getting the ideal solutions, the next step is to calculate the distance of the alternatives from (A^*) and (A^-) using Equation (6) and (7), respectively.

Step 7: After calculating the range of alternatives to (A^*) and (A^-), the next step is to obtain the correlation coefficients between the three alternatives. The calculation is done using Equation (8). The results are shown in Table X.

TABLE X
THE CORRELATION COEFFICIENT OF EACH ALTERNATIVE

Alternative	*Correlation Coefficient*
Main Board	*0.67*
Second Board	*0.62*
MESDAQ Market	*0.63*

Based on Table X, it can be seen that the correlation coefficients of the first alternative, namely the Main Board is of the highest value followed by MESDAQ Market and the Second Board. Correlation coefficients for the Main

Board is of *0.67*, while the Second Board and MESDAQ Market, each has value *0.62* and *0.63*. Based on the correlation coefficients, an alternative to selecting the firms listed on the Main Board should be the first choice, followed by selecting the firms listed on the MESDAQ Market and the last one is to select a firm on the Second Board.

In essence, the greater the value of the correlation coefficient indicates the priorities of the decision to be made. This method not only allows the decision maker to provide the rank of each alternative, but also shows the degree of likelihood of alternative selection as illustrated in Table X. It should be noted that our results are based out of the three criteria set out earlier this analysis (market valuation, stock trading volume and stock trading value).

From Table X, it is also apparent that the correlation coefficients for the Second Board and MESDAQ only differ by 0.01. However, although the difference is only one percent, the result is significant for the decision makers in determining the order of the ranking. Therefore, the implementation of fuzzy TOPSIS in this scenario is really effective in real world applications. The proposed method is very appropriate when dealing with subjective assessment of the real environment that is full of uncertainties.

VI. CONCLUSION

In this paper, fuzzy TOPSIS was applied in the selection of the best investment boards according to three criteria by incorporating operational risks in investment. First criteria is market valuation, second criteria is stock trading volume and third criteria is stock trading value. Results obtained from the relative closeness to the ideal solutions were used to rank the preference order in the selection of investment boards for stock exchanges. Clearly, the application of fuzzy set theory in conjunction with TOPSIS is effective in order to provide a more realistic solution to the process of decision making in stock investment.

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