

## Documents

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**On the Dynamics of Geometric Quadratic Stochastic Operator Generated by 2-Partition on Countable State Space**  
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### Abstract

A quadratic stochastic operator (QSO) is frequently acknowledged as the analysis source to investigate dynamical properties and modeling in numerous areas. Countless classes of QSO have been investigated since the operator was introduced in 1920s. The study of QSOs is still an open problem in the nonlinear operator theory field, especially QSOs on infinite state space. We are interested in the dynamics of Geometric QSO generated by a 2-partition defined on countable state space. We first show the system of equations formed from defined Geometric QSO with infinite-dimensional space can be simplified into a one-dimensional setting, corresponding to the number of defined partitions. The trajectory behavior of such a system is investigated by using functional analysis approach, where the operator either converges to a unique fixed point or has a second-order cycle. It is shown that such an operator can be either regular or nonregular for arbitrary initial points depending on the value of parameters. In this research, we present two cases, i.e., two different parameters and three different parameters. We display the form of the fixed point and periodic points of period-2. Moreover, an example for the nonregular transformation will be provided, where such QSO has 2-periodic points. © 2022, Malaysian Journal of Mathematical Sciences. All Rights Reserved.

### Author Keywords

Countable set; Geometric distribution; Measurable partition; Quadratic stochastic operator; Regularity

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