

The Analytic Hierarchy Process: A Tool for Prioritization, Evaluation, and Selection



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Presentation Outline

- Introduction
- Illustration of AHP through examples
 - Consistency
 - Combining group judgments
- AHP in Management Research

Introduction

- ❑ Multiple Criteria Decision Making Method
- ❑ Introduced by Prof. Thomas L. Saaty in 1977 (University of Pennsylvania)
- ❑ Research
 - Theory
 - Applications
- ❑ Fuzzy AHP
- ❑ International symposium on the AHP
- ❑ International Journal of the AHP
- ❑ Analytic Network Process (ANP)

Areas of AHP Applications

- Accounting
- Banking & Finance
- Conflict Analysis
- Energy Planning
- Education
- Environmental Management
- Forecasting
- Healthcare
- Human Resource Management
- Information Systems
- Marketing
- Military
- Operations Management
- Politics
- Portfolio Management
- Project Management
- R & D Management
- Resource Allocation
- Risk Analysis
- Sports
- Strategic Management
- Technology
- Total Quality Management
- Transportation

AHP Research Objectives

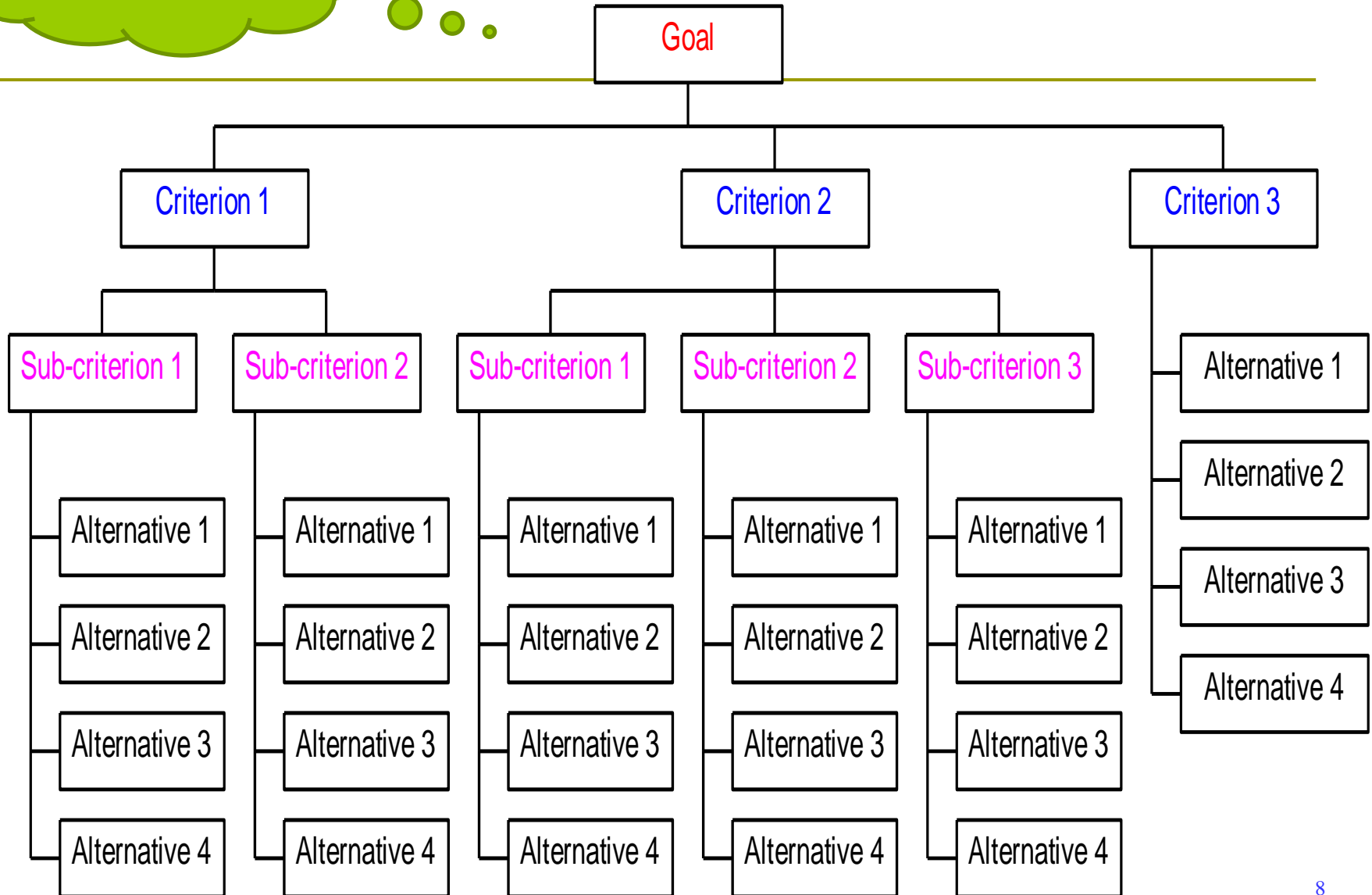
- To prioritize a set of factors/criteria/attributes
 - Executive selection criteria
 - Benchmark firm evaluation criteria
- To select the best option/alternative from a number of
 - Best project
 - Best company to partner with
- To rank a set of alternatives
 - University ranking
 - Country ranking w.r.to quality of life, etc
- To evaluate a set of alternatives/To measure performance
 - Employee performance evaluation
 - Organizations performance evaluation

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- To develop an index
 - Competitiveness
 - Green application
 - Shari'ah application index
 - Banks performance index

Analytic Hierarchy Process:

Relative Measurement

Hierarchy



AHP Method:

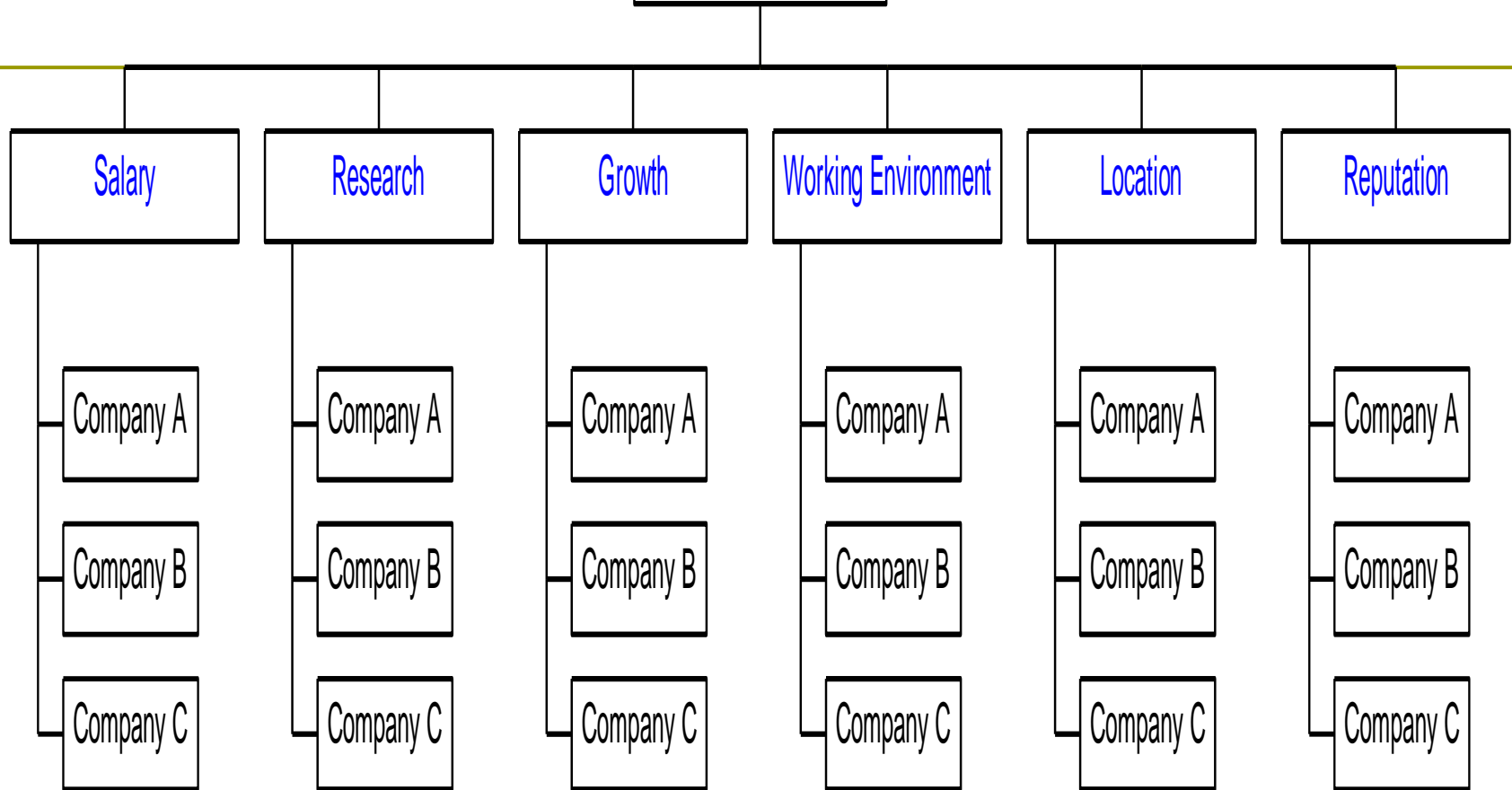
- Step 1:** Decompose the problem and identify the **criteria** and **alternatives**. Construct the **hierarchy**.
- Step 2:** Construct **pairwise comparison matrices** for all the criteria and alternatives.
- Step 3:** Determine the **weights** of the criteria and **local weights** of the alternatives from the above matrices by using a **suitable weight determination technique**.
- Step 4:** Obtain the **overall weights** of the alternatives by synthesizing the local weights.

Example (Job Selection)

Suppose, after graduation, in order to get a job, Razali has appeared in a number of interviews. Ultimately, he has been offered by three companies, say Company A, Company B and Company C. The problem now before Razali is to select (or decide) the company where he will join.

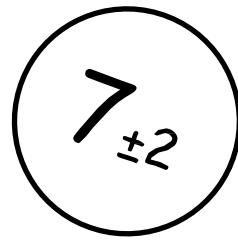
Six criteria: 1) Salary, 2) Research, 3) Growth, 4) Working Environment, 5) Location, and 6) Reputation. Discuss how to use AHP to assist Razali in guiding him to select the best job.

Selection of the best job



AHP Ratio Scale

Verbal judgment of importance	Rating
Equally important	1
Equally to moderately important	2
Moderately important	3
Moderately to strongly important	4
Strongly important	5
Strongly to very strongly important	6
Very strongly important	7
Very strongly to extremely important	8
Extremely important	9
Note: $a_{ji} = 1/a_{ij}$	



Justification of 1-9 scale

- ❑ The capacity of human short term memory is seven separate items, plus or minus two.
- ❑ The brain of a regular human can simultaneously process, differentiate, and deal with at most 7 factors
- ❑ For some people this limit can be decreased to 5, for some other people it can be increased to 9

George A. Miller

"The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information"

•(The Psychological Review, 1956, vol. 63, pp. 81-97)

General Form of a Pairwise Comparison Matrix

$A =$

	C_1	C_2	C_n
C_1	a_{11}	a_{12}	a_{1n}
C_2	a_{21}	a_{22}	a_{2n}
C_n	a_{n1}	a_{n2}	a_{nn}

Pairwise comparisons

□ Salary

- Research
- Growth
- WE
- Location
- Reputation

□ Research

- Growth
- WE
- Location
- Reputation

Pairwise comparisons

- Growth
 - WE
 - Location
 - Reputation
- WE
 - Location
 - Reputation
- Location
 - Reputation

Salary is

- equally important with research (1)
- equally important with growth (1)
- moderately to strongly important compared to working environment (4)
- equally important with location (1)
- equally to moderately **less** important than reputation (1/2)

Research is

- equally to moderately important compared to growth (2)
- moderately to strongly important compared to working environment (4)
- equally important with location (1)
- equally to moderately less important than reputation (1/2)

Growth is

- ❑ Strongly more important compared to working environment(5)
- ❑ Moderately more important compared to location(3)
- ❑ Moderately less important than reputation(1/2)

Working Environment is

- ❑ Moderately less important than location (1/3)
- ❑ Moderately less important than reputation (1/3)

Location is

- ▣ Equally important with reputation (1)

Criteria Pairwise Comparison Matrix (Upper Triangular Part)

	SAL	RES	GRO	WEN	LOC	REP
SAL		1	1	4	1	$\frac{1}{2}$
RES			2	4	1	$\frac{1}{2}$
GRO				5	3	$\frac{1}{2}$
WEN					$\frac{1}{3}$	$\frac{1}{3}$
LOC						1
REP						

English Premier League results

Team	Arsenal	Aston Villa	Chelsea	Everton	Leicester city	Liverpool
Arsenal		3-0	0-1	2-1	2-1	0-0
Aston Villa	0-3		0-3	1-1	1-1	0-6
Chelsea	1-0	3-0		3-0	2-0	1-3
Everton	1-2	1-1	0-3		2-3	1-1
Leicester city	1-2	1-1	0-2	3-2		2-0
Liverpool	0-0	6-0	3-1	1-1	0-2	

Criteria Pairwise Comparison Matrix

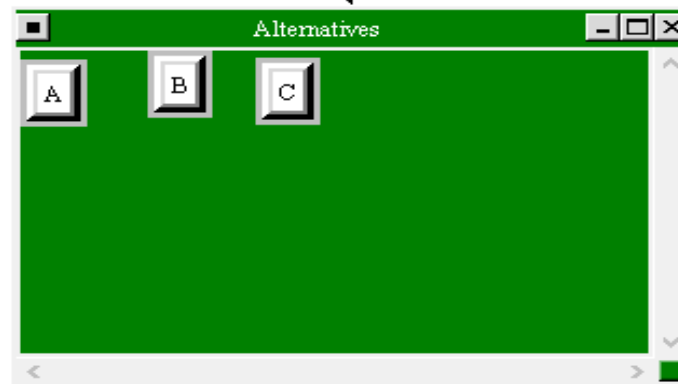
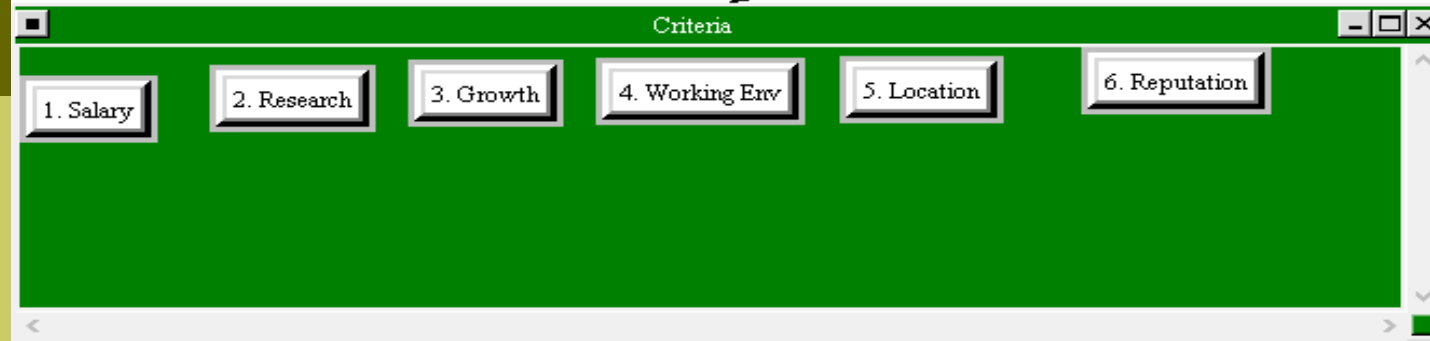
	SAL	RES	GRO	WEN	LOC	REP
SAL	1	1	1	4	1	$\frac{1}{2}$
RES	1	1	2	4	1	$\frac{1}{2}$
GRO	1	$\frac{1}{2}$	1	5	3	$\frac{1}{2}$
WEN	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{5}$	1	$\frac{1}{3}$	$\frac{1}{3}$
LOC	1	1	$\frac{1}{3}$	3	1	1
REP	2	2	2	3	1	1

Priority Extraction Methods

- Eigenvector Method
- Logarithmic Least Squares Method (Geometric Mean method)
- Least Squares Method
- Mathematical Programming Method
- Row-Column Normalisation Method

AHP Software

- ❑ Superdecision (www.superdecisions.com)
- ❑ Expertchoice (www.expertchoice.com)
- ❑ Excel adds-in (<http://bpmsg.com/ahp-excel-template/>)



**Superdecision
Interface**

Criteria Pairwise Comparison Matrix with weights

	SAL	RES	GRO	WEN	LOC	REP	Weights
SAL	1	1	1	4	1	1/2	0.159
RES	1	1	2	4	1	1/2	0.184
GRO	1	1/2	1	5	3	1/2	0.198
WEN	1/4	1/4	1/5	1	1/3	1/3	0.049
LOC	1	1	1/3	3	1	1	0.155
REP	2	2	2	3	1	1	0.253

Measuring Consistency

$$CI = \frac{\lambda - n}{n - 1} = \frac{6.425 - 6}{6 - 1} = 0.085$$

Random Index for various sizes of PCM

Size of PCM	1	2	3	4	5	6	7	8
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41

Size of PCM	9	10	11	12	13	14	15
RI	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Consistency Ratio

$$CR = \frac{CI}{RI} = \frac{0.085}{1.24} = 0.0685$$



AHP Ratio Scale

Verbal judgment of preference	Rating
Equally preferred	1
Equally to moderately preferred	2
Moderately preferred	3
Moderately to strongly preferred	4
Strongly preferred	5
Strongly to very strongly preferred	6
Very strongly preferred	7
Very strongly to extremely preferred	8
Extremely preferred	9
Note: $a_{ji} = 1/a_{ij}$	

PCM for Salary

SAL	A	B	C	W _t
A	1	1/4	1/2	0.14
B	4	1	3	0.63
C	2	1/3	1	0.24

PCM for Research

RES	A	B	C	W _t
A	1	1/4	1/5	0.10
B	4	1	1/2	0.33
C	5	2	1	0.57

PCM for Growth

GRO	A	B	C	Wt
A	1	3	1/3	0.32
B	1/3	1	1	0.24
C	3	1	1	0.44

PCM for Working Environment

WEN	A	B	C	W _t
A	1	1/3	5	0.28
B	3	1	7	0.64
C	1/5	1/7	1	0.07

PCM for Location

LOC	A	B	C	W _t
A	1	1	7	0.47
B	1	1	7	0.47
C	1/7	1/7	1	0.07

PCM for Reputation

REP	A	B	C	Wt
A	1	7	9	0.75
B	1/7	1	5	0.19
C	1/9	1/5	1	0.06

Synthesis

	p₁	p₂	...	p_m
A ₁	q ₁₁	q ₂₁	...	q _{m1}
A ₂	q ₁₂	q ₂₂	...	q _{m2}
...
A _n	q _{1n}	q _{2n}	...	q _{mn}

$$w_j = \sum_{i=1}^m p_i q_{ij}, \quad j=1,2,\dots,n$$

Obtaining the global weights

	SAL (0.16)	RES (0.19)	GRO (0.20)	WEN (0.05)	LOC (0.15)	REP (0.26)	Overall Weights
A	0.14	0.10	0.32	0.28	0.47	0.75	0.38
B	0.63	0.33	0.24	0.64	0.47	0.19	0.36
C	0.24	0.57	0.44	0.07	0.07	0.06	0.26

Principle of hierarchical
composition

Example 2 (Nuclear Waste Disposal)

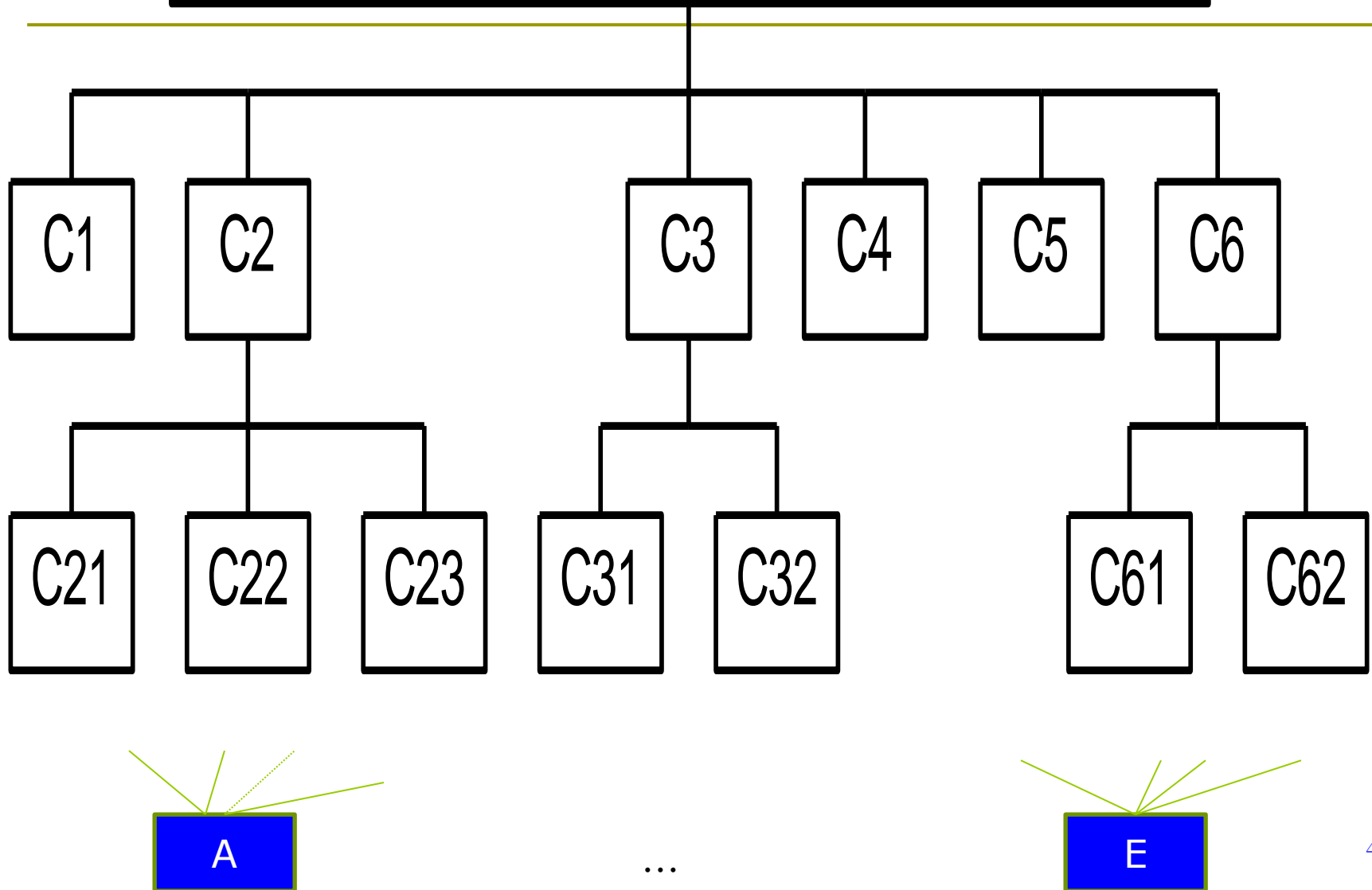
Suppose Government of India wants to determine the best strategy for high level nuclear waste disposal. The strategies are:

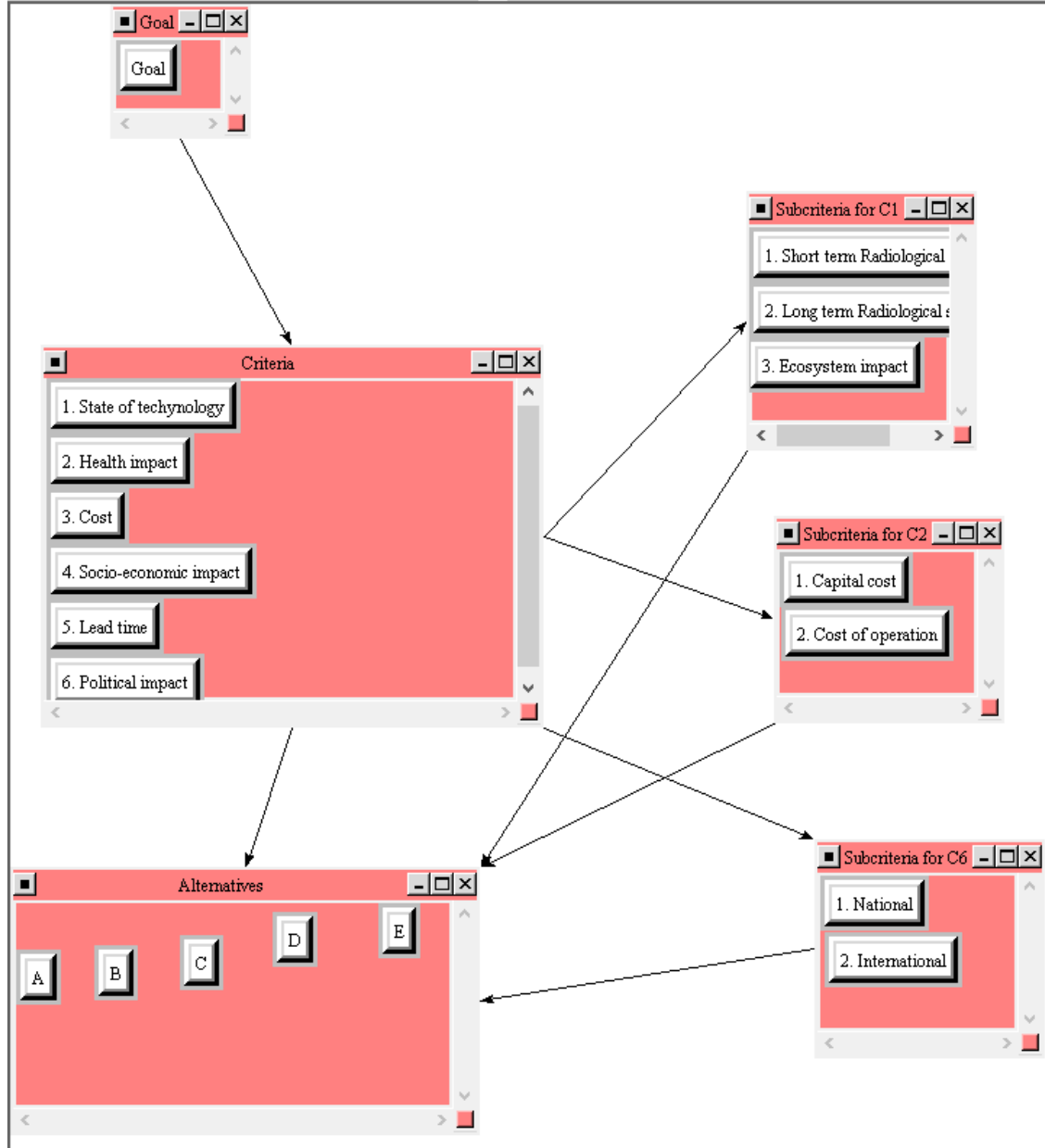
1. Geological disposal (A)
2. Very deep hole (B)
3. Island disposal (C)
4. Subseabed disposal (D)
5. Disposal into space (E)

Suppose you are the consultant, guide the government in recommending the best option.

No.	Criteria	Sub-criteria
1.	State of technology (C_1)	
2.	Health, safety and environmental impacts (C_2)	i) Short-term radiological safety (C_{21}) ii) Long-term radiological safety (C_{22}) iii) Ecosystem impacts (C_{23})
3.	Cost (C_3)	i) Capital cost (C_{31}) ii) Cost of operation (C_{32})
4.	Socio-economic impact (C_4)	
5.	Lead time (C_5)	
6.	Political impact (C_6)	i) National (C_{61}) ii) International (C_{62})

Selecting the best nuclear waste disposal strategy





PCM to Determine Criteria Weights

	C_1	C_2	C_3	C_4	C_5	C_6	Wts.
C_1	1	1	5	7	5	5	0.35
C_2		1	5	7	5	5	0.35
C_3			1	5	2	2	0.11
C_4				1	1/4	1/5	0.03
C_5					1	1	0.07
C_6						1	0.08
CR = 0.05							

PCM for sub-criteria of C_2

C_2	C_{21}	C_{22}	C_{23}	Wts.
C_{21}	1	1/3	1/5	0.10
C_{22}		1	1/3	0.26
C_{23}			1	0.64

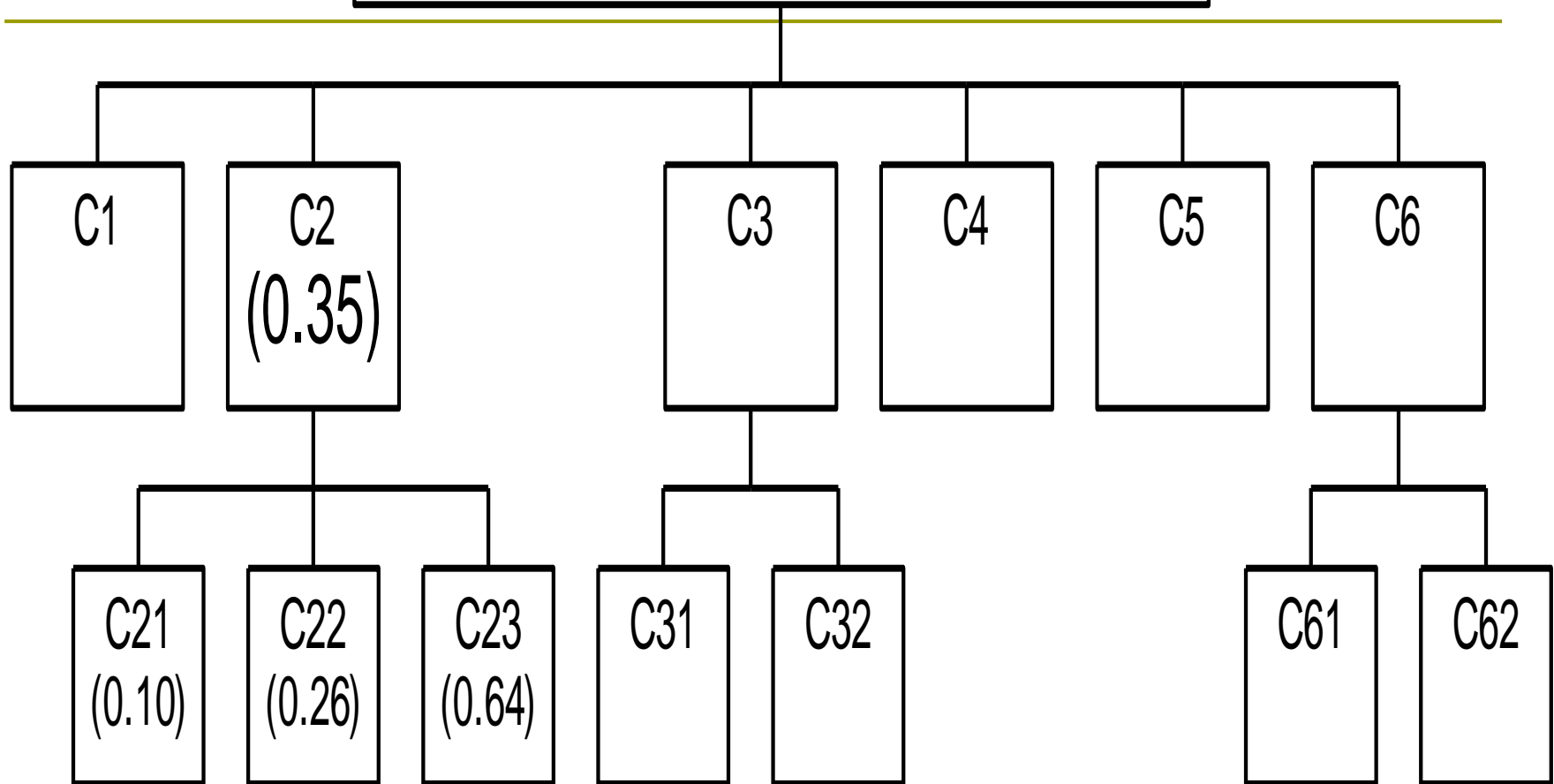
PCM for sub-criteria of C_3

C_3	C_{31}	C_{32}	Weights
C_{31}	1	5	0.83
C_{32}	1/5	1	0.17
CR = 0.00			

PCM for sub-criteria of C_6

C_6	C_{61}	C_{62}	Weights
C_{61}	1	$1/2$	0.33
C_{62}	2	1	0.67
CR = 0.00			

Selecting the best nuclear waste disposal strategy



No.	Criteria and Sub-criteria	Weight
1	State of technology	0.35
2	Health, safety and environment impact -Short- term radiological safety -Long-term radiological safety -Ecosystem impact	0.35 $(.35 \times .10) = .035$ $(.35 \times .26) = .09$ $(.35 \times .64) = .22$
3	Cost -Capital cost -Cost of operation	0.11 $(.11 \times .83) = .09$ $(.11 \times .17) = .02$
4	Socio-economic impact	.03
5	Lead time	.07
6	Political impact -national international	.08 $(.08 \times .33) = 0.03$ $(.08 \times .67) = 0.05$

PCM for Alternatives (C_1)

C1	A	B	C	D	E	Wts.
A	1	5	3	4	8	0.487
B		1	1/3	1/2	5	0.099
C			1	2	7	0.235
D				1	5	0.144
E					1	0.034

PCM for Alternatives (C₂₁)

C21	A	B	C	D	E	Wts.
A	1	2	3	2	7	0.388
B		1	2	1	6	0.222
C			1	1/2	5	0.132
D				1	6	0.222
E					1	0.038

PCM for Alternatives (C₂₂)

C22	A	B	C	D	E	Wts.
A	1	1/3	3	1/3	1/5	0.085
B		1	5	2	1/3	0.232
C			1	1/5	1/7	0.042
D				1	1/3	0.176
E					1	0.465

PCM for Alternatives (C₂₃)

C23	A	B	C	D	E	Wts.
A	1	1	3	2	1/3	0.184
B		1	3	2	1/3	0.184
C			1	1/2	1/5	0.066
D				1	1/4	0.106
E					1	0.459

PCM for Alternatives (C31)

C31	A	B	C	D	E	Wts.
A	1	1/5	5	1/4	1/7	0.067
B		1	7	3	1/3	0.256
C			1	1/7	1/9	0.028
D				1	1/5	0.147
E					1	0.500

PCM for Alternatives (C₃₂)

C32	A	B	C	D	E	Wts.
A	1	1/5	5	1/7	1/6	0.068
B		1	7	1/3	1/2	0.189
C			1	1/9	1/7	0.029
D				1	3	0.467
E					1	0.255

PCM for Alternatives (C₄)

C4	A	B	C	D	E	Wts.
A	1	1/3	5	1/2	1/5	0.097
B		1	7	3	1/3	0.250
C			1	1/5	1/7	0.035
D				1	1/5	0.127
E					1	0.490

PCM for Alternatives (C₅)

C5	A	B	C	D	E	Wts.
A	1	3	2	4	5	0.412
B		1	1/2	2	3	0.155
C			1	4	5	0.285
D				1	2	0.090
E					1	0.058

PCM for Alternatives (C₆₁)

C61	A	B	C	D	E	Wts.
A	1	1/3	1	1/4	1/5	0.068
B		1	3	1/2	1/3	0.169
C			1	1/4	1/5	0.068
D				1	1/2	0.270
E					1	0.425

PCM for Alternatives (C₆₂)

C62	A	B	C	D	E	Wts.
A	1	1	1	7	7	0.304
B		1	1	7	7	0.304
C			1	7	7	0.304
D				1	1	0.043
E					1	0.043

Synthesis

	C1 0.35	C21 .035	C22 .09	C23 .22	C31 .09	C32 .02	C4 0.03	C5 0.07	C61 0.03	C62 0.05	Glob al
A	0.487	0.388	0.085	0.184	0.067	0.068	0.097	0.412	0.068	0.304	0.294
B	0.099	0.222	0.232	0.184	0.256	0.189	0.250	0.155	0.169	0.304	0.172
C	0.235	0.132	0.042	0.066	0.028	0.029	0.035	0.285	0.068	0.304	0.149
D	0.144	0.222	0.176	0.106	0.147	0.467	0.127	0.090	0.270	0.043	0.141
E	0.034	0.038	0.465	0.459	0.500	0.255	0.490	0.058	0.425	0.043	0.242

Global (Overall) Weights of the Alternatives

Alternative	Global Weight
A : Geological Disposal	0.294
B: Very Deep Hole	0.172
C: Island Disposal	0.149
D: Subseabed Disposal	0.141
E: Disposal into Space	0.242

Group Decision Making

Respondent 1: "State of Technology" is strongly more important than "H-S-E"

• Respondent 2 "H-S-E" is strongly more important than "State of Technology"

	S-o-T	H-S-E
S-o-T	1	5
H-S-E		1

• Combined Decision?

	S-o-T	H-S-E
S-o-T	1	1/5
H-S-E		1

	S-o-T	H-S-E
S-o-T	1	?
H-S-E	1/?	1

Group Decision Making

•DM1
•very strongly
(7)

	A	B
A	1	X
B	1/X	1

DM2
moderately (3)

•Use arithmetic mean
to combine decisions?

NO

$$\begin{aligned} \bullet X &= (7+3)/2 = 5 \\ \bullet 1/X &= (1/7+1/3)/2 = 0.24 \neq 1/5 \end{aligned}$$

•Use geometric mean
to combine decisions?

YES

$$\begin{aligned} \bullet X &= \sqrt{7*3} = 4.58 \\ \bullet 1/X &= \sqrt{1/7*1/3} = 0.22 = 1/4.58 \end{aligned}$$

AHP Geometric Mean calculation using Microsoft Excel

PART 2

AHP in Research

AHP in Management Research

- ❑ Operations Management
- ❑ Environmental Management
- ❑ Strategic Management
- ❑ Total Quality Management
- ❑ Information Systems Management
- ❑ Human Resource Management
- ❑ Marketing Management

[File 1](#)

[File 2](#)

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- ❑ Critical Success factors of Malaysian Vision 2020
 - ❑ Dimensions of Quality in Healthcare Sector
 - ❑ Excellence Management Model for Hotel Industry

Superdecisions Video Links

No.	Topic	URL
1	Simple AHP without subcriteria	https://youtu.be/hjRxht__2wI
2	With subcriteria	https://youtu.be/HwkyhIiG8V0
3	Sensitivity analysis	https://youtu.be/v7UDNvciQ38
4	Complete example	https://youtu.be/cstrfin0bYo
5	Ratings mode	https://youtu.be/1ZqR-wGKHGQ

YouTube Search Keywords: AHP Superdecisions software for Beginners

Concluding Remarks

- ❑ AHP is a highly successful decision making tool. The method has been widely applied in various disciplines
- ❑ Simplicity and mathematical rigour are behind its popularity
- ❑ Analytic Network Process is the generalised version of AHP
- ❑ Many research avenues are still open on theoretical development of AHP as well as its applications

