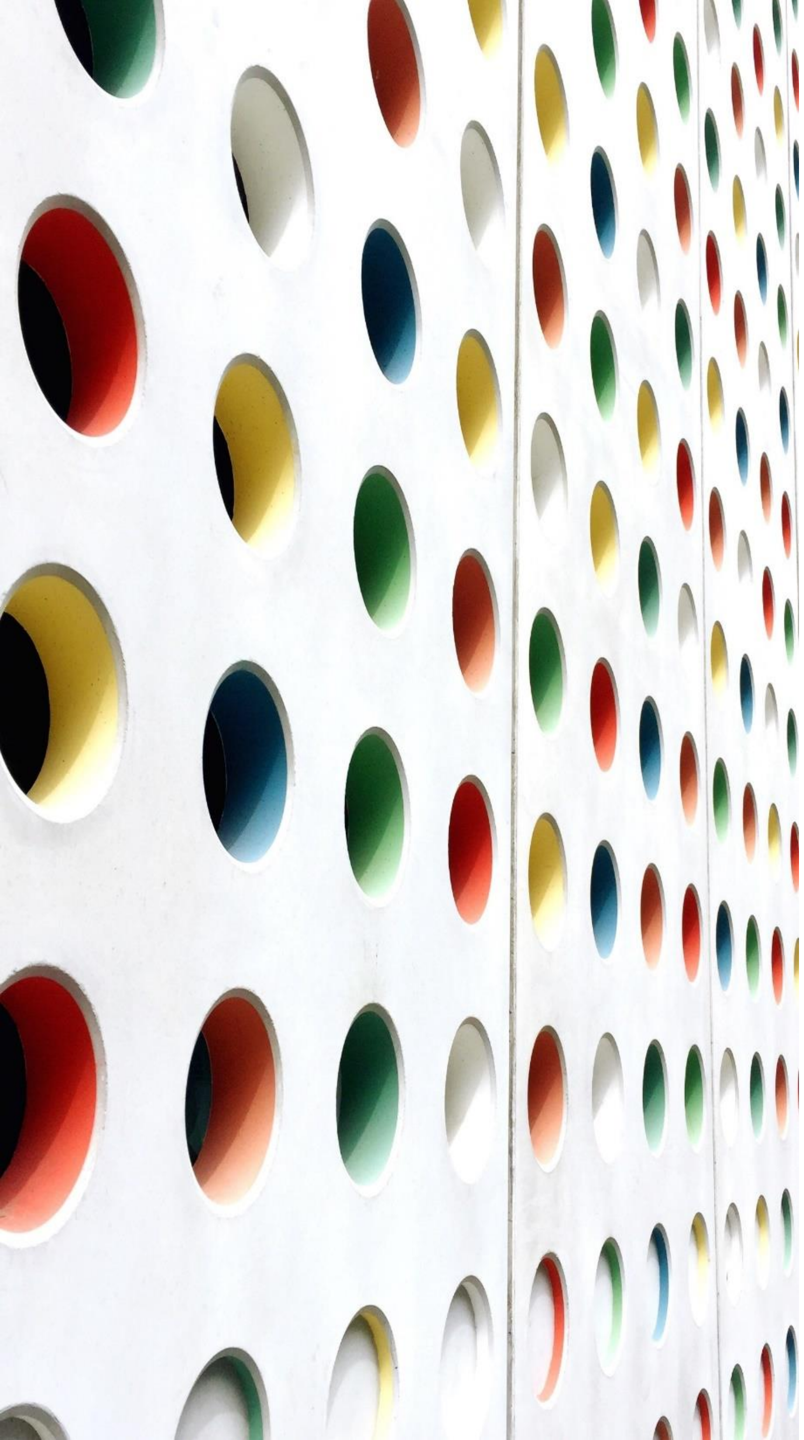

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THE APPROXIMATION OF THE SOLUTION OF HEAT CONDUCTION PROBLEM IN CIRCULAR PLATE WITH CONCENTRATED INITIAL HEAT

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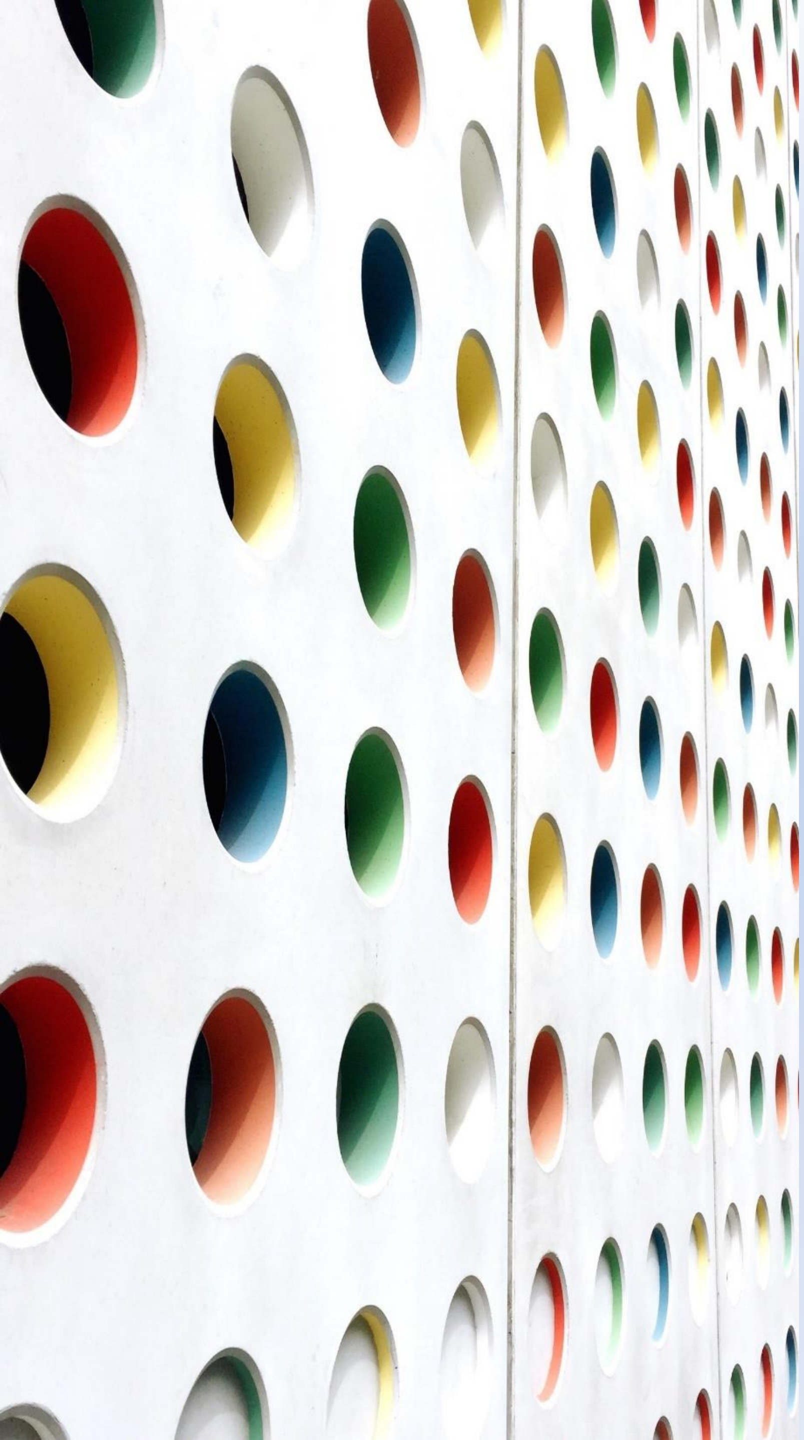
CONTENT

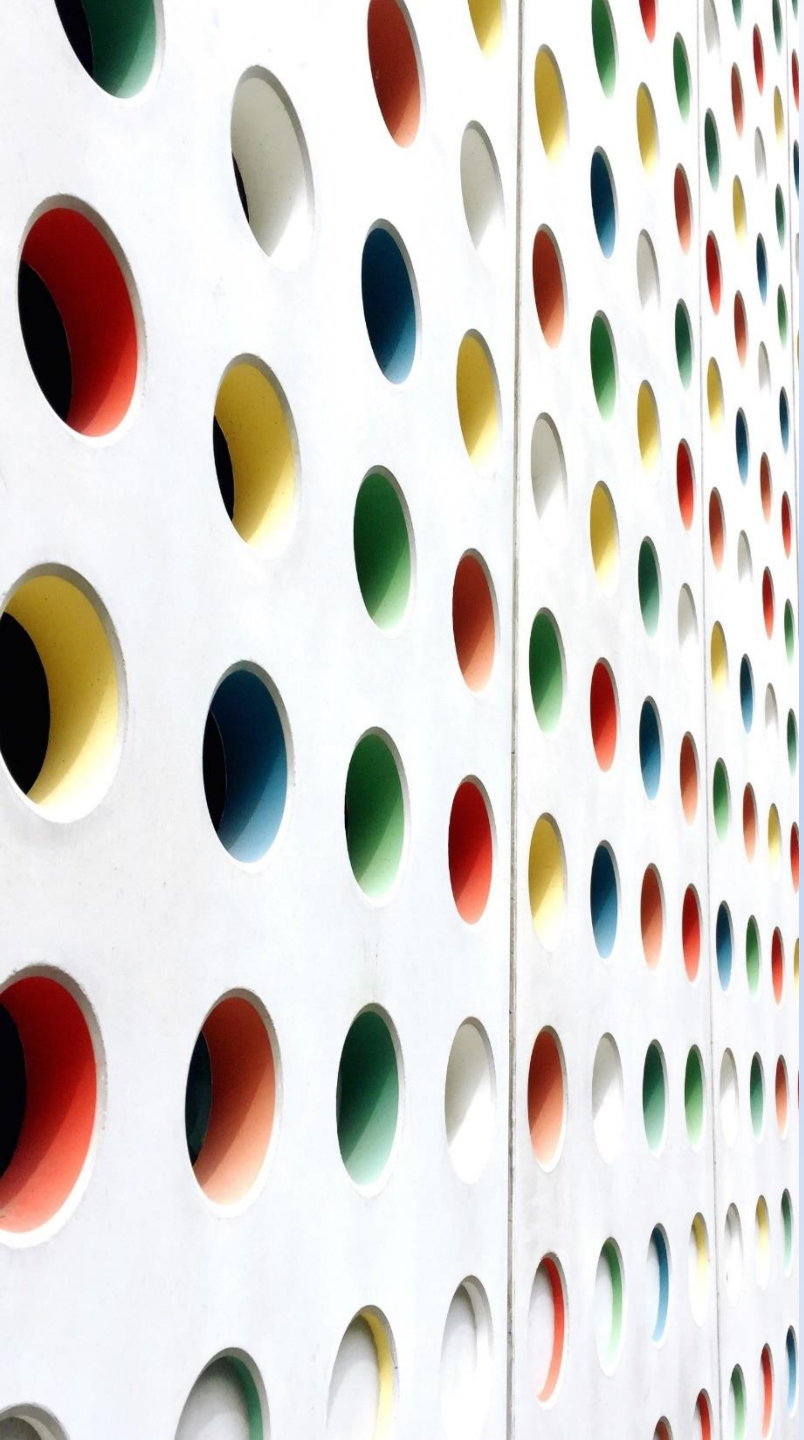


INTRODUCTION

I. INTRODUCTION

Many problems of the engineering sciences can be solved using the modern methods of equations of mathematical physics, particularly elliptic differential equations play essential role in solving heat and mass transfer problems in engineering processes. In this paper, a numerical approximation of the solution of heat conduction problem in circular plate with initial concentrated heat is constructed by using the Riesz means of the spectral decompositions. Solution of heat transfer problems are subjected to the distributional boundary conditions and initial conditions.

When an engineering process occur on the plate, the solution of the equations of mathematical physics can be solved by separation method but spectral expansions of the solution does not converge to the boundary function. This difficulty can be overcome by regularization of the spectral expansions. Regularization of the divergent series solution is accurate numerical interpretations of the solutions of the problems.



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LITERATURE REVIEW

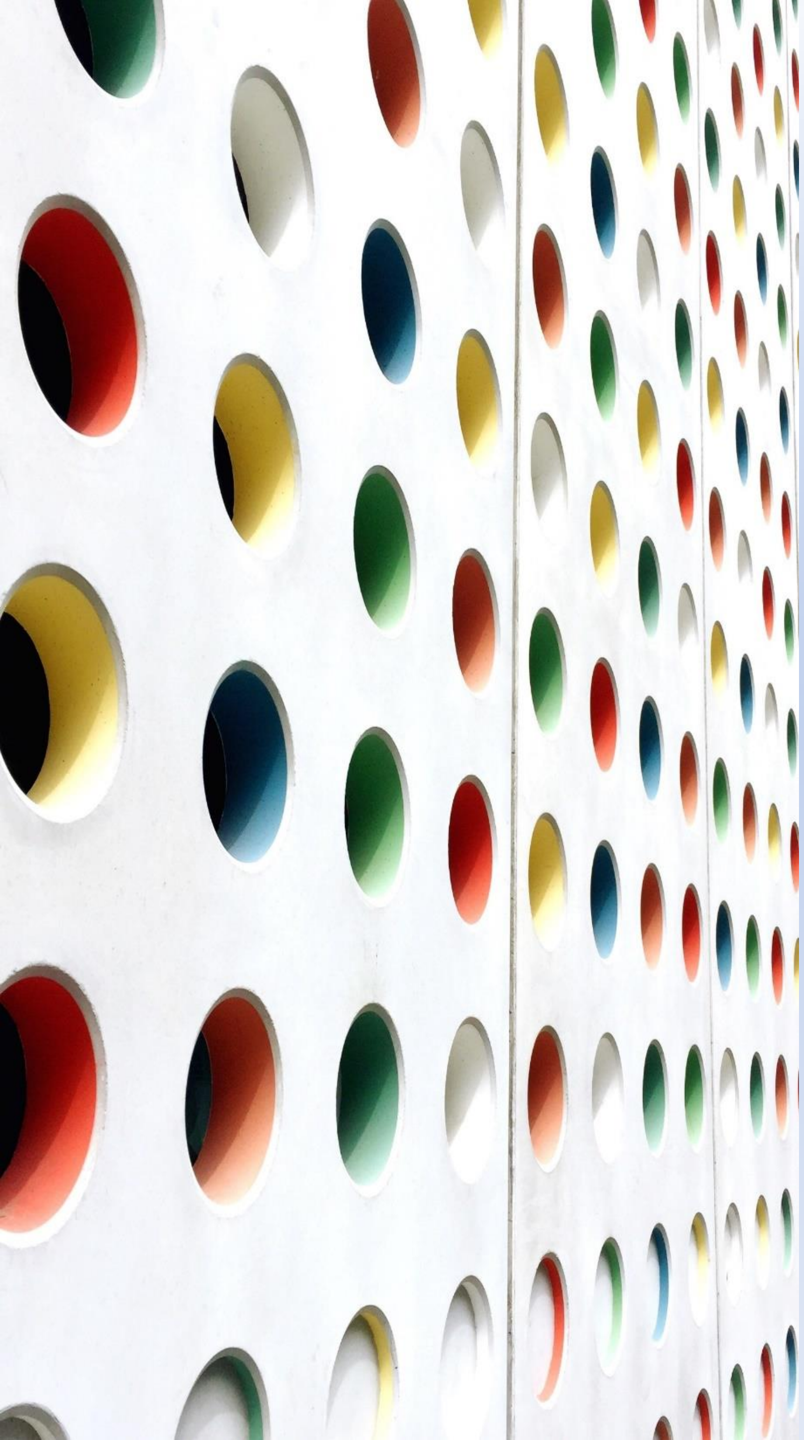
- Spectral expansions of the distributions related to the Schrodinger operator was investigated in [1].
- The optimization of the regularization of the Fourier series corresponding to the distributions defined in the plate of vibration problem studied in [2], where the regularization of the spectral decomposition of the distributions is used to obtain the solutions of the plate vibration problem at a fixed point of the plate with the initial time and critical index of the Riesz means.

The work [3] is devoted to the study on the approximations of distributions on a smooth manifold, where author has obtained the sufficient conditions for summability of the spectral decompositions corresponding to the distributions in weak topology. Localization problems of the spectral decompositions connected with periodic distributions are considered in [4] (Abdulkasim Akhmedov et al 2021 J. Phys.: Conf. Ser. 1988 012085), where authors obtained sufficient conditions for the equiconvergence of the spectral decompositions of the distributions connected with the elliptic differential operator on the torus with Fourier integral in the classes of the Sobolev.

Precise conditions of the summability and localization of the spectral expansions associated with various partial differential operators are discussed in A.Rakhimov (2019) Localization of the Spectral Expansions Associated with the Partial Differential Operators, Mathematical Methods in Engineering, pp.217-233. The authors studied the problems in the spaces of both smooth functions and singular distributions related to spectral expansions of elliptic operators in the Sobolev spaces. All theorems are formulated in terms of the smoothness and degree of the regularizations.

Further results in latter expanded to the more general spectral expansions in (Alimov & Rakhimov, Localization of Spectral Expansions of Distributions, 1996), (Rakhimov & Alimov, Localization of Spectral Expansions of Distributions in a Closed Domain, 1997), (Rakhimov, On the Localization of Multiple Trigonometric Series of Distributions, 2000), (Rakhimov, Localization Conditions for Spectral Decompositions Related to Elliptic Operators from Class, 1996), (Rakhimov, Ahmedov, & Hishamuddin, On the Spectral Expansions of Distributions Connected with Schrodinger Operator, 2012), (Rakhimov, Spectral Decompositions of Distributions from Negative Sobolev Classes, 1996), (Rakhimov, On the uniform convergence of Fourier series on a closed domain, 2017), (Rakhimov, On the uniform convergence of Fourier series, 2016). We note that the results on the summability of the spectral decomposition connected with Fourier-Laplace series are obtained in (Ahmedov, Nurullah & Rakhimov, 2013).

In this paper, a numerical approximation of the solution of heat conduction problem in circular plate with initial concentrated heat is constructed by using the Riesz means of the spectral decompositions. Solution of heat transfer problems are subjected to the distributional boundary conditions and initial conditions.



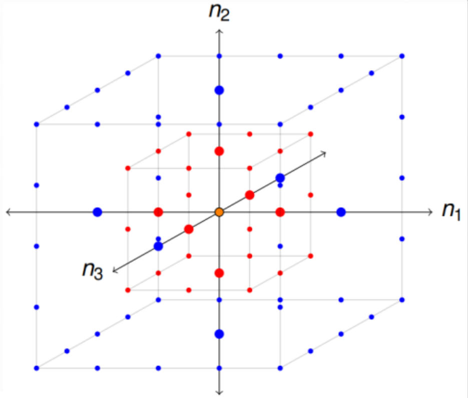
METHODOLOGY

I. Multiple Trigonometric Series

The basic objects of my study are functions of several variables which are 2π - periodic in each of these variables. Let consider a fundamental set, where our functions are defined, to be the N-dimensional cube

$$\mathbb{T}^N = \{\mathbf{x} \in \mathbb{R}^N : -\pi < x_j \leq \pi, j = 1, \dots, N\}$$

This cube can be identified in a natural way with an N-dimensional torus (i.e. the subset of $\mathbb{C}^N$ of the form $\left(e^{ix_1}, e^{ix_2}, \dots, e^{ix_N}\right)$, where $(x_1, x_2, \dots, x_N) \in \mathbb{R}^N$, which in what follows we shall not distinguish from $\mathbb{T}^N$.



The fundamental harmonic with respect to which we take our expansion have in appearance exactly the same form as in the one-dimensional case: $e^{i\mathbf{n}\cdot\mathbf{x}}$ where, $N > 1$, $\mathbf{n}\cdot\mathbf{x}$ stands for the inner product

$$\mathbf{n}\cdot\mathbf{x} = n_1x_1 + \cdots + n_Nx_N, \mathbf{n} \in \mathbb{Z}^N, \mathbf{x} \in \mathbb{R}^N.$$

Here, as usual, $\mathbb{Z}^N$ is the set of all vectors with integer components: $\mathbf{n} = (n_1, \dots, n_N)$ is in $\mathbb{Z}^N$ if $n_j = 0, \pm 1, \dots$.

A *multiple trigonometric series*

$$\sum_{\mathbf{n}\in\mathbb{Z}^N} c_n e^{i\mathbf{n}\cdot\mathbf{x}} \qquad (1)$$

where the coefficients c_n are arbitrary complex numbers, looks exactly as an ordinary "one-dimensional" series.

The *circular partial sum* (or *spherical* if $N \geq 3$) $\tilde{S}_R(x)$ takes the form

$$\tilde{S}_R(\mathbf{x}) = \sum_{|\mathbf{n}| \leq R} c_n e^{i\mathbf{n}\cdot\mathbf{x}} \tag{8}$$

The series (1) is *circularly convergent* (or spherically if $N \geq 3$) if $\lim_{R\to\infty} \tilde{S}_R(x)$ exists.

Forms of Convergence.

The most important forms of convergence are uniform convergence, convergence at a fixed point, a. e. convergence, convergence in the metric $L_p(\mathbb{T}^N)$. Let us assume that the series (1) converges in a determined sense to a function $f(x)$:

$$f(\mathbf{x}) = \sum_{\mathbf{n}\in\mathbb{Z}^N} c_n e^{i\mathbf{n}\cdot\mathbf{x}} \tag{11}$$

So far, we have not put any restrictions whatsoever on the coefficients c_n, which may have been arbitrary complex numbers. One of the first problems which arises in the sturdy of orthogonal series (to which also trigonometric series belong) is the problem of determining the coefficients.

The solution to this problem depends on in which sense the series converges to f. If the type of convergence allows one to integrate the series term by term (for example, in the case of uniform convergence or convergence in L_p), then upon multiplying both members of (11) with $e^{-i\mathbf{n}\cdot\mathbf{x}}$ and integrating over $\mathbb{T}^N$ we obtain $c_n = f_n$, where the numbers

$$f_n = (2\pi)^{-N} \int_{\mathbb{T}^N} f(x) e^{-i\mathbf{n}\cdot\mathbf{x}}\, d\mathbf{x} \tag{12}$$

are referred to as the *Fourier coefficients* of f. It is clear that the Fourier coefficients exist only if $f \in L_1(\mathbb{T}^N)$, i. e. if f is a function summable over $\mathbb{T}^N$ (with respect to the N-dimensional Lebesgue measure). The Fourier coefficients f_n of a function $f \in L_1(\mathbb{T}^N)$ tend to zero as $|n| \to \infty$ (*Riemann Lebesgue theorem*).

With each function $f \in L_1(\mathbb{T}^N)$ we can thus associate a multiple trigonometric series

$$f(\mathbf{x}) \sim \sum_{\mathbf{n} \in \mathbb{Z}^N} f_n e^{i\mathbf{n}\cdot\mathbf{x}} \tag{12}$$

termed the *Fourier series of* f. The sign ~ means that the series has been obtained in a purely formal way without any statements about its convergence. The following important problem arises now: must the Fourier series converge in some sense, and if this is the case, does it converge to the function f?

2. DISTRIBUTIONS ON $\mathbb{T}^N$

We deal with the spectral expansions of the functions defined on the torus $\mathbb{T}^n$, which can be defined by $\mathbb{T}^n = (\mathbb{R}/2\pi\mathbb{Z})^n = \mathbb{R}^n/2\pi\mathbb{Z}^n$. The torus $\mathbb{T}^n$ often is identified with the cube $[-\pi, \pi)^n \subset \mathbb{R}^n$, where the measure on the torus is identified with the restriction of the Euclidean measure on the cube. Functions on $\mathbb{T}^n$ are the functions on $\mathbb{R}^n$ that are 2π-periodic in each of the coordinates.

We may identify functions on $\mathbb{T}^n$ with $2\pi\mathbb{Z}^n$ −periodic functions on $\mathbb{R}^n$.

Let $\mathcal{F}_{\mathbb{R}^n}$ be the Euclidian Fourier transforms defined by

$$(\mathcal{F}_{\mathbb{R}^n})f(\boldsymbol{\xi}) := (2\pi)^{-N} \int_{\mathbb{R}^n} e^{-i\mathbf{x}\cdot\boldsymbol{\xi}} f(\mathbf{x})d\mathbf{x}.$$

and its inverse $\mathcal{F}_{\mathbb{R}^n}^{-1}$ is given by

$$f(\mathbf{x}) = \int_{\mathbb{R}^n} e^{-i\mathbf{x}\cdot\boldsymbol{\xi}} (\mathcal{F}_{\mathbb{R}^n} f)(\boldsymbol{\xi})d\boldsymbol{\xi},$$

Let

$$\mathcal{F}_{\mathbb{T}^n} = (f \mapsto \hat{f}) : C^{\infty}(\mathbb{T}^n) \to S(\mathbb{Z}^n)$$

by the toroidal Fourier transform defined by

$$\hat{f}(\mathbf{m}) := (2\pi)^{-N} \int_{\mathbb{T}^n} e^{-i\mathbf{x}\cdot\mathbf{m}} f(\mathbf{x})d\mathbf{x}.$$

Space $L^2(\mathbb{T}^n)$ is a Hilbert space with the inner product

$$(u, v)_{L^2(\mathbb{T}^n)} := \int_{\mathbb{T}^n} u(\mathbf{x})\overline{v(\mathbf{x})}d\mathbf{x},$$

where $\bar{z}$ is the complex conjugate of $z \in \mathbb{C}$.

The Fourier coefficients of $u \in L^2(\mathbb{T}^n)$ are

$$\hat{u}(\boldsymbol{m}) = (2\pi)^{-N} \int_{\mathbb{T}^n} e^{-i\boldsymbol{m}\cdot\mathbf{x}} u(\boldsymbol{x}) d\mathbf{x}, \qquad \mathbf{m} \in \mathbb{Z}^n$$

and they are well-defined for all $\mathbf{m} \in \mathbb{Z}^n$ due to Hölder's inequality and compactness of $\mathbb{T}^n$.

The system of functions $\{e^{i\mathbf{m}\cdot\mathbf{x}} : \mathbf{m} \in \mathbb{Z}^n\}$ forms an orthonormal basis on $L^2(\mathbb{T}^n)$. Thus the partial sums of the Fourier series

$$\sum_{\mathbf{m}\in\mathbb{Z}^n} \hat{u}(\mathbf{m}) e^{i\mathbf{m}\cdot\mathbf{x}}$$

converge to u in the L^2 −norm, so that we shall identify u with its Fourier representation:

$$u(\mathbf{x}) = \sum_{\mathbf{m}\in\mathbb{Z}^n} \hat{u}(\mathbf{m}) e^{i\mathbf{m}\cdot\mathbf{x}}$$

3. SPECTRAL DECOMPOSITIONS.

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In order to make the presentation as transparent as possible, we will begin our discussion with the classical Laplace operator

$$\Delta = \sum_{k=1}^{N} \frac{\partial^2}{\partial x_k^2}.$$

The operator Δ is considered in the Hilbert space $L_2(\mathbb{T}^N)$as an unbounded operator with domain of definition $c^\infty(\mathbb{T}^N)$.

As for any two functions u and v in the domain of definition holds,

$$(\Delta u, v) = (u, \Delta v)$$

$$(\Delta u, u) = -(\nabla u, \nabla u) \leq 0$$

The operator $-\Delta$ is symmetric and nonnegative. Consequently, by Friedrichs's theorem (*) it has a nonnegative self-adjoint extension, which we denote by $\hat{A}$. It is not hard to see that this self-adjoint extension is unique and coincides with the closure of $-\Delta$.

(*) (cf. Alimov, Il'in, and Nikishin (1976/77))

The operator $\hat{A}$ has in $L_2(\mathbb{T}^N)$ a complete orthonormal system of eigenfunctions

$$\{(2\pi)^{-N/2}e^{inx}\}, \qquad n \in \mathbb{Z}^N,$$

corresponding to the eigenvalues $\{|n|^2\},\ \ n \in \mathbb{Z}^N$. Like every self-adjoint operator, the extension $\hat{A}$ has in view of von Neumann's theorem a decomposition of unity $\{E_\lambda\}$ with the aid of which it can be written in the form (cf. Alimov, Il'in, and Nikishin (1976/77))

$$\hat{A} = \int_0^\infty \lambda dE_\lambda\,.$$

It is easy to check that the operators E_λ have the form

$$E_\lambda f(x) = \sum_{|n|^2<\lambda} f_n e^{inx}\,,$$

where f_n are the Fourier coefficients of the function $f \in L_2(\mathbb{T}^N)$ defined with the help of the identity (12). As we have seen, the family $\{E_\lambda f\}$ called the *spectral expansion* of f, coincides with the spherical partial sums of the Fourier series (13).

Let us now consider instead of the Laplace operator an arbitrary differential operator with constant coefficients

$$(D) = \sum_{|\boldsymbol{\alpha}|\leq m} a_{\boldsymbol{\alpha}} D^{\boldsymbol{\alpha}} \qquad \textbf{(14)}$$

where $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \dots, \alpha_N\}$ is a multi-index, i.e. an element of $\mathbb{Z}^N$ with non-negative coordinates, $|\boldsymbol{\alpha}| = \alpha_1 + \alpha_2 \cdots + \alpha_N$, $D^{\boldsymbol{\alpha}} = D_1^{\alpha_1} D_2^{\alpha_2} \cdots D_N^{\alpha_N}$, $D_j = \frac{1}{i}\frac{\partial}{\partial x_j}$

We associate with each differential operator (14) its symbol, an algebraic polynomial in N variables:

$$A(\boldsymbol{\xi}) = \sum_{|\boldsymbol{\alpha}|\leq m} a_{\alpha} \boldsymbol{\xi}^{\boldsymbol{\alpha}}$$

where $\boldsymbol{\xi} = (\xi_1, \xi_2, \dots, \xi_N)$, $\boldsymbol{\xi}^{\boldsymbol{\alpha}} = \xi_1^{\alpha_1} \xi_2^{\alpha_2} \cdots \xi_N^{\alpha_N}$.

A differential operator is called elliptic if its principal symbol

$$A_0(\boldsymbol{\xi}) = \sum_{|\boldsymbol{\alpha}|=m} a_{\boldsymbol{\alpha}} \boldsymbol{\xi}^{\boldsymbol{\alpha}} \quad \text{(14)}$$

is positive definite, i.e., if for any $\boldsymbol{\xi} \in \mathbb{R}^N, \boldsymbol{\xi} \neq \mathbf{0}$, we have $A_0(\boldsymbol{\xi}) > 0$.

If the coefficients of the operator (14) are real, then $A(D)$ is symmetric, i.e.,

$$(Au, v) = (u, Av), \qquad u, v \in C^\infty(\mathbb{T}^N).$$

If $A(D)$ is also elliptic then by the well-known Garding's inequality (cf. Hormander (1983-85), Vol. III) it is bounded from below

$$(Au, u) \geq c(u, u), c \in \mathbb{R}^1.$$

Therefore, by Friedrichs's theorem just mentioned, $A(D)$ has a self-adjoint extension $\hat{A}$ and

$$\hat{A} = \int_{c}^{\infty} \lambda dE_{\lambda}$$

where $\{E_{\lambda}\}$ is the corresponding resolution of identity. As in the case of the Laplace operator, the eigenfunctions of $\hat{A}$ are the functions $(2\pi)^{-N/2}e^{inx}$ while the eigenvalues equal $A(n)$.

The spectral expansion of a function $f \in L_2(\mathbb{T}^N)$ takes the form

$$E_{\lambda}(A)f(\mathbf{x}) = \sum_{A(\mathbf{n})<\lambda} f_n e^{i\mathbf{n}\cdot\mathbf{x}} \quad (16)$$

We say that the spectral expansion (16) is summable to $f(x)$ by the Riesz method of order s if

$$\lim_{\lambda\to\infty} E_\lambda^s f(\mathbf{x}) = f(\mathbf{x}).$$

Let us note that the integral in (17) makes sense for any real $s \geq 0$ and even for complex s with $Re\ s \geq 0$, thanks to which one can use interpolation theorems in the study of Riesz means.

If A is elliptic operator then $E_\lambda f$ is given by (16). Inserting this into (17) gives

$$E_\lambda^s f(\mathbf{x}) = \sum_{A(\mathbf{n})<\lambda} \left(1 - \frac{A(\mathbf{n})}{\lambda}\right)^s f_n e^{i\mathbf{n}\cdot\mathbf{x}} \qquad (18)$$

- NUMERICAL APPROXIMATATION BY SPECTRAL DECOMPOSITIONS

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As a result, we obtain the solution of the wave problem

$$u_t = k\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right), \qquad 0 < r < 1, 0 \le \theta \le \frac{\pi}{2}$$

with the given boundary conditions

$$u(1,\theta,t) = 0, \qquad 0 \le \theta \le \frac{\pi}{2}$$

and initial conditions

$$u(r,0,t) = 0, u_\theta\left(r,\frac{\pi}{2},t\right) = 0, 0 < r < 1$$

$$u(r,\theta,t) \text{ bounded}$$

$$u(r,\theta,0) = f(r,\theta)$$

- Separation of variables proceeds as follows.
- $u = T(t)\Theta(\theta)R(r)$
- Substituting this into the equation in (2.1), we have
- $\frac{T'(t)}{kT(t)} = \frac{\Theta(\theta)R''(r)+\frac{1}{r}\Theta(\theta)R'^{(r)}+\frac{1}{r^2}\Theta''(\theta)R(r)}{\Theta(\theta)R(r)} = -\lambda^2$

- This leads to the following equations:
- $T'(t) + k\lambda^2 T(t) = 0$
- $r^2 R'' + rR' + (\lambda^2 r^2 - \mu^2)R = 0$
- $\Theta^2 + \mu^2 \Theta = 0$
- with boundary conditions
- $|R(0)| < \infty,\ R(1) = 0$

- $\Theta(0) = 0,\ \Theta'\left(\frac{\pi}{2}\right) = 0.$
- First, we consider the equation for Θ:
- $\Theta^2 + \mu^2\Theta = 0,\ \ \Theta(0) = 0,\ \Theta'\left(\frac{\pi}{2}\right) = 0$
- which gives
- $\mu_m = (2m - 1),\ T_m(t) = \Theta_{\mathrm{m}}(\theta) = \frac{2}{\sqrt{\pi}}\sin\big((2m-1)\theta\big)\,.$
- The eigenfunction $\Theta_m(\theta)$ satisfy the orthogonality relations
- $\int_0^{\pi/2} \Theta_m(\theta)\Theta_k(\theta)\,d\theta = \delta_{m,k}.$

$$E_\lambda^s u(r,\theta,t) =$$

$$\frac{2}{\sqrt{\pi}} \sum_{m^2+n^2<\lambda^2} \sum \left(1 - \frac{m^2+n^2}{\lambda^2}\right)^s \kappa_{m,n} C_{m,n} e^{-k\lambda_{m,n}^2 t} J_{2m-1}\left(\lambda_{m,n} r\right) \sin\left((2m-1)\theta\right)$$

Table 1 Solution of the regularized series for different values of s ($s = 0, s = 1, s = 2$), $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$ and $t = 1$.

Result from MATLAB

λ	$E_\lambda^0 u_t(\pi/4, \pi/4, 0)$	$E_\lambda^1 u_t(\pi/4, \pi/4, 0)$	$E_\lambda^2 u_t(\pi/4, \pi/4, 0)$
850	7.00000000000111	-0.00411526008122465	0.00315287084334459
2000	7.05258074162887e-12	0.00437619755418917	0.00224139519878630
2500	-5.99999999999709	-0.00424875153686566	0.00139881698091759
3500	-11.9999999999978	0.00265330270521225	0.000670850331949691
4000	-1.49999999998671	-0.00301893616894222	0.000496940178414063
6000	-14.4999999999866	0.00390290242268027	0.000192221516156804
6500	8.00000000000999	-0.00459805620623655	0.000158396465790134
7000	4.50000000002167	0.00370161678701597	0.000129784596000590
8000	17.5000000000110	-0.00293904530899636	0.0000919562578425550
8500	-21.9999999999889	0.00475047959718943	0.0000777342813546000
9000	9.99999999997983	-0.00392974892915876	0.0000669218950743560

Table 2 Solution of the regularized series for different values of s ($s = 1.4$, $s = 1.5$, $s = 1.6$), $x = \frac{\pi}{4}$, $y = \frac{\pi}{4}$ and $t = 1$.

Result from MATLAB

λ	$E_{\lambda}^{1.4} \frac{\partial}{\partial t} u(\pi/4, \pi/4, 0)$	$E_{\lambda}^{1.5} \frac{\partial}{\partial t} u(\pi/4, \pi/4, 0)$	$E_{\lambda}^{1.6} \frac{\partial}{\partial t} u(\pi/4, \pi/4, 0)$
850	0.00803593046444506	-0.00115524562312308	0.00142023240665223
2000	0.000875761946008247	0.000963512443736381	0.00112013058002347
2500	-0.000784349026601738	-0.000780214420703640	0.000834852932308561
3500	-0.000499871318010436	0.000780214420703640	0.000476428457911737
4000	-0.00204231619672313	-0.000286399500720402	0.00033969719526239
6000	-0.000187415490236209	0.000177104174169329	0.000175328908221639
6500	0.000152049739436416	-0.000152735642328927	0.000152411243847878
7000	0.000248924682223761	0.000767207453907792	0.000102762495466868
8000	0.00508716709242962	-0.000739754672066731	0.0000843421787644696
8500	-0.000859976145705169	0.000796355406602819	0.0000775983948437184
9000	0.000776983513026165	-0.000771428936626843	0.0000731753987851646

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❖ CONCLUSIONS

The solution of the vibration problems had a form of double Fourier series and it requires some regularizations. Based on the singularity, we considered the Reisz method of summation as regularization of the Fourier series solutions of the wave problems. To minimize the length of numerical calculations of the wave problems, we optimize the regularized Fourier series solutions.

To optimize the regularization of the solutions of the plate vibration problems we found the minimum order of the Reisz means. The minimum order was $s > \frac{N-1}{2} - 1$. Therefore, a programming (MatLab) is used to obtain the numerical solutions. We found the optimization of the regularization of the series solutions at a fixed point of the plates at initial time and critical index. As it is estimated, we achieved the good convergence after critical point. Moreover, we studied optimization of the regularization of the Fourier series of rectangular plate vibration problem and a numerical method is used to find the series solution at a fixed point of the plate at initial time and critical index. Numerical approximations of the solution of the wave equation are constructed to verify the convergence theorem.

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